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VI. *On the Regularity in the Distribution of the Satellites of Spectrum Lines.* By PROF. H. NAGAOKA, *Imperial University, Tokyo.*

RECEIVED SEPTEMBER 4, 1916.

§ 1. The nature of the satellites of spectrum lines is not yet thoroughly known, nor are the positions with respect to the principal well determined, except for a few lines of a small number of elements. I have no doubt that there exists regularity in the distribution of the satellites, in their order of intensity, and in their behaviour when acted upon by strong electric and magnetic forces. The usual view as regards the satellites is to consider them as belonging to the principal line, which is the strongest among an assemblage of lines of various strengths clustered together within a narrow compass about the principal. It is still an open question whether such grouping of lines is swayed by the principal, or they have independent existence apart from the principal. There is as yet no observation of the effect of an electric field on the satellites, and on account of the difficulty of measurement and of the complexity of the separation the Zeeman effect is known only for a limited number of satellites. Experiments on the effect of the magnetic field on the satellites of mercury lines show that some of them are controlled by the principal or strong satellites in the neighbourhood, but there exist satellites which seem to lead an independent existence, the mode of magnetic separation being little affected by the principal. It may, therefore, be improper to call them satellites, while some of them are strongly influenced and bear the character of the name assigned to them.

§ 2. In a former communication to the Society,* we have noticed that the satellites of mercury lines are apparently distributed with some regularity and that there is gradation in intensity. Although no definite result could then be obtained, it was probable that the minute structure of the principal lines was not yet known, as most of the discrepancies among different observations were to be traced to the inaccuracy in the determination of the exact position of the principal lines. As suggested by Stansfield, the position of the satellites referred to one of the distinct components gave tolerably good concordance in the results of different observers with various instruments, and it seemed quite likely that further study of the principal line would reveal the regularity in the distribution of the satellites, if such really exists.

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* Nagaoka and Takamine, "Proc." Phy. Soc., 25, p. 1, 1912.

§ 3. For this purpose, it is necessary to use instruments with high resolving power, in order that a strong line with rather vague boundaries may be separated into its components. A concave grating is generally inefficient, the echelon grating and Lummer-Gehrcke plate have definite resolving power for a given wave-length, so that we can not bring different powers to bear upon the lines to be analysed. The inconvenience is especially felt when the interval between successive spectra is so narrow that the lines are superposed and cause great confusion in discriminating them. The disadvantage arising from the definiteness of the resolving power can in no wise be remedied, so that it is generally expedient to resort to those interferometers in which the thickness of the air space contributing to interference can be easily changed, so as to suit the purpose of the experiment. The interferometers of Michelson and of Fabry-Perot belong to this type of instruments, as the air interspace is adjustable within wide limits. A simple device for changing the resolving power is a prime factor that makes the instrument indispensable in researches relating to the minute structure of a spectrum line, especially when it consists of a cluster of lines within narrow limits. The easiest way of arriving at a desired resolving power would be to increase the air interspace between the plates in a Fabry-Perot interferometer, the thickness being, of course, within the limit of interference for the line to be examined. The exact value of the resolving power cannot, however, be given, as it depends on the number of reflections from the semi-transparent silver films. One serious objection to a thick interspace is the crowding of the lines, as different interference fringes are intermixed with each other, and hence arises the trouble of discriminating the different lines, especially when there are numerous satellites. In fact, the appearance of interference rings is extremely vague, when the interspace is several centimetres thick, that it is almost impossible to tell even the position of the strong satellites. This difficulty can, however, be to a great extent overcome by the use of interference points, obtained by crossing it with another instrument, which may have either fixed or adjustable resolving power. The advantage* gained by crossing is especially great, when there is ambiguity in the position of the satellites as observed with a single instrument; for it can easily be settled by examining the crossed spectra. With an adjustable interferometer, we can change the air interspace, such that the satellites to be analysed are brought distinctly under observation, without being intermixed with other components. I have tried the combinations of F.P. interferometer with echelon grating, L.G. plate, wedge plate, and another F.P. interferometer of the fixed type; the last method of crossing was found to have a disadvantage of losing too much light in passing through four semi-transparent silver films, so that most of the measurements were made with the two other combinations. In addition to this, I have found that the combination of echelon grating or L.G. plate with a wedge plate forms an excellent pair for work with fixed resolving power.

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* Nagaoka and Takamine, "Phil. Mag.," 27, p. 126, 1914.

§ 4. Preliminary study of the minute structure of mercury lines was made in conjunction with Mr. Takamine,* and published in connection with the Zeeman effect of the mercury line λ 4,359; but subsequent experiments showed that there were many points still to be improved in the method of investigation, especially in increasing the order of interference, for making the fine structure of the principal line easily visible on the photograms.

§ 5. It was first shown by Janicki† that the principal line of the green line of mercury is multiple, by combining wedge and L.G. plates, which had a resolving power of 680,000 for the said line. Although Janicki denies the multiple character of other lines of mercury, it seemed quite likely that fine structure of the principal lines could not be placed under a strict experimental test, inasmuch as the resolving power of his instrument was fixed. With sliding F.P. interferometer, the resolving power can be raised to several times the above value, and the structure more accurately analysed.

§ 6. In the present experiment, the air interspace was much greater than in the former and generally ranged between 6 cm. and 12 cm.; with the thickness of 7 cm. or 8 cm., the composite character of the principal of the green line was distinctly visible, but owing to the overlapping of different spectra, it was necessary to cross it with a L.G. plate whose $d\lambda_{max.}=0{\cdot}1066$ A.U.; the resolving power thus utilised may be estimated at $2{\cdot}3\times 10^6$ for 8 cm. air space, assuming the number of reflections on the front face to be 8. The spectra were drawn out more than with the echelon grating; but, owing to the crowding of interference points resulting from numerous satellites, it was somewhat difficult to discriminate them; from our previous knowledge of the position of the satellites, we can, however, avoid their coincidence with the principal line, by calculating the value of the air interspace suitable for the experiment. In addition to this, the intensity of the principal line was far greater than that of the satellites, so that there was no trouble in following the curves of interference arising from the components of the principal line.

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* Nagaoka and Takamine, "Phil. Mag.," 29, p. 241, 1915.

† Janicki, "Ann. d. Phys.," 33, p. 438, 1912.

§ 7. The mercury lamp was of the Arons-Lummer type, water-cooled, and run with a current of 2·5 to 3 amperes at 100 volts; in the circuit was included a big electromagnet having sufficient inductance to maintain the current steadily. Great care was taken to keep the temperature of the air interspace constant; the resistance and other parts, which are liable to be heated, were kept outside the room. The interferometer was enclosed in a wooden box, leaving only a passage for light and photographic apparatus. Wratten-Wainright panchromatic plates were generally used for the green and yellow lines, and process plates for the violet lines. The interference points were measured by means of a micrometer made by Repsold, or with a dividing engine constructed by Société Genèvoise, having attached microscopes and an arrangement for eliminating screw error.

§ 8. It will be necessary to give a short explanation of the locus of interference points obtained by the combination of interferometers above mentioned. For the F.P. instrument, the order of interference P for light of wave-length λ subtending a ring of small angle 2θ is

$$P=\frac{2e\cos\theta}{\lambda}=\frac{2e}{\lambda}\left(1-\frac{\theta^2}{2}\right),$$

e denoting the thickness of the air interspace. Referred to rectangular co-ordinates x and y through the centre of the ring in the focal plane of the observing telescope, and supposing that the wave-length λ is different by a small amount $\delta\lambda$ from λ_0, we get

$$\frac{x^2+y^2}{\text{[illegible]}}=1-\frac{P}{2e}(\lambda_0+\delta\lambda)=\left(1-\frac{P}{P_0}\right)-\frac{P}{P_0}\frac{\delta\lambda}{\lambda_0},$$

where P_0 stands for $2e/\lambda_0$.

Since P is nearly equal to P_0 and very large,

$$\frac{x^2+y^2}{2}=\frac{\delta P}{P_0}-\frac{\delta\lambda}{\lambda},$$

where $\delta P=P_0-P$.

The order h for L.G. plate is given by

$$h=\frac{2t\sqrt{(\mu^2-\sin^2 i)}}{\lambda},$$

where t is the thickness, μ the index of refraction for light of wave-length λ, and i the angle of exit. As observations are generally made near grazing exit, we can put

$$i=90^\circ-\alpha$$

where α is small. Thus,

$$h^2\lambda^2=t^2\{4(\mu^2-1)+2\alpha^2\}.$$

Putting $\lambda=\lambda_0+\delta\lambda$, where λ_0 is the wave-length of the reference line, and $\delta\lambda$ is the difference in wave-length of the satellites from it, we get

$$2h^2\lambda_0\delta\lambda=\{4t^2(\mu^2-1)-h^2\lambda^2\}+2t^2\alpha^2,$$

the quantity within the parenthesis being constant.

§ 9. On combining the plate with F.P. interferometer, the centre of the ring does not always coincide with the direction of grazing incidence; we have, therefore, to put $\alpha=a+x$; consequently $2h^2\lambda_0\delta\lambda=2h_0^2\lambda_0\delta\lambda+2t^2(a+x)^2$ nearly.

Equating $\delta\lambda$ in these two equations for F.P. and L.G. interferometers, and using the relation $\frac{4t^2}{h_0^2\lambda_0^2}=\frac{1}{\mu^2-1}$, we get the equation

$$x^2+\frac{(a+x)^2}{2(\mu^2-1)}+y^2=2\left(\frac{\delta P}{P_0}-\frac{\delta h}{h_0}\right),$$

which is evidently an equation to an ellipse. Thus the interference points to be obtained by the said combination lie on ellipses with the centre on the x-axis. If we now adjust the position of the plate, such that the ray at grazing exit passes through the centre of the rings, $a=0$, and the equation reduces to

$$\frac{2\mu^2-1}{2(\mu^2-1)}x^2+y^2=2\left(\frac{\delta P}{P_0}-\frac{\delta h}{h_0}\right).$$

The major axis of the ellipse is evidently on the y-axis, and the excentricity$=1/\sqrt{(2\mu^2-1)}$. In the present experiment for λ 5,461, the eccentricity amounts to $1/1{\cdot}994$ and the ratio of the axes 1·156, being nearly$=\sqrt{4/3}$. For the same value of δP, the axes of the ellipse become smaller as the air interspace is increased, so that the ellipses for successive orders of P become denser, till they are obliterated by coming into contact, as the limit of interference is reached. With the green and the violet line of mercury, the interference is still visible for $e=12$ cm., corresponding to the values of $P=440,000$ and $550,000$ respectively. From the measurements of the breadth of interference points, I found the order at the limit of interference to be 750,000 and 810,000 respectively, nearly coinciding with the values found by Fabry and Perot by another method.

§ 10. For a given wave-length, the value of h in the L.G. plate is fixed, as the angular interval of the successive spectra cannot be changed. With F.P. interferometer, this is nevertheless the case. With increased air space, the interval for the same value of P decreases, as the angular difference for successive spectra is given by the relation $\Delta\theta=-\frac{\lambda}{2e\theta}=-\frac{1}{P_0\theta}$. Consequently the value of $\frac{\delta P}{P_0}-\frac{\delta h}{h_0}$ becomes smaller as the interspace is widened. As the axes of the ellipse on which the interference points for the same orders of P and h lie are determined by the above term, the curve for the same difference of P and h occupies a smaller space as the resolving power of the interferometer is increased. The experiments in this direction indicate that the disadvantage thus caused is of little consequence, provided the temperature of the interspace does not vary during observation or photographing.

§ 11. Another important question in connection with the present combination is the variation of x and y co-ordinates, as we pass from one interference spectrum to that succeeding it. For simplicity's sake, I shall make the supposition that the ray at grazing incidence passes through the centre of the rings, *i.e.*, $a=0$. Keeping λ constant,

$$h^2-h_0^2=\frac{2t^2}{\lambda^2}x^2$$

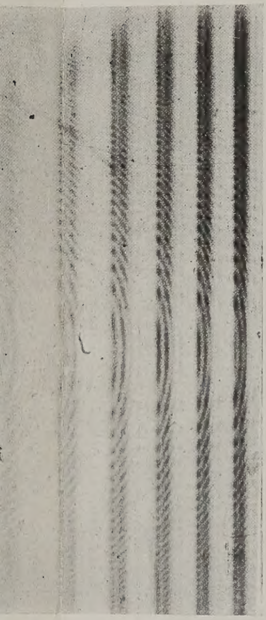
for L.G. plate and $$P-P_0=-\frac{e}{\lambda}(x^2+y^2)$$

for F.P. interferometer. For finite differences of h and P, we get approximately

$$\lambda^2 h\Delta h=2t^2x\Delta x$$

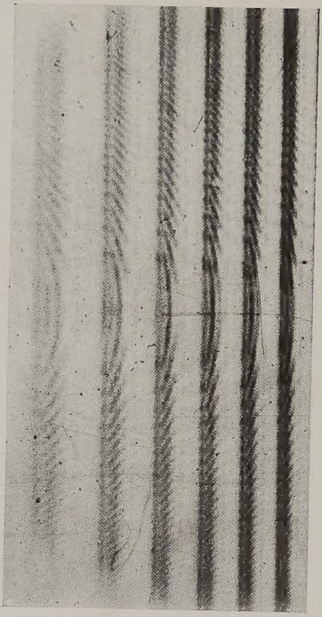
and $$\lambda\Delta P=-2e(x\Delta x+y\Delta y).$$

Fig. 1a.
λ 5,461.



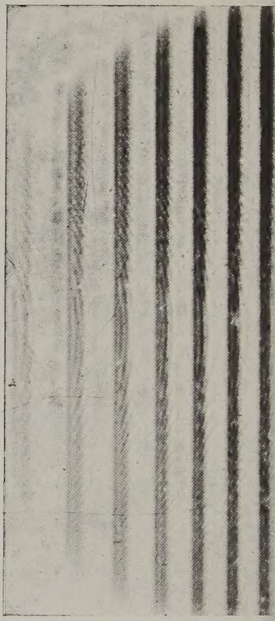
$e=8{\cdot}8$ *cm.*
L.G.+F.P.

Fig. 1b.
λ 5,461.



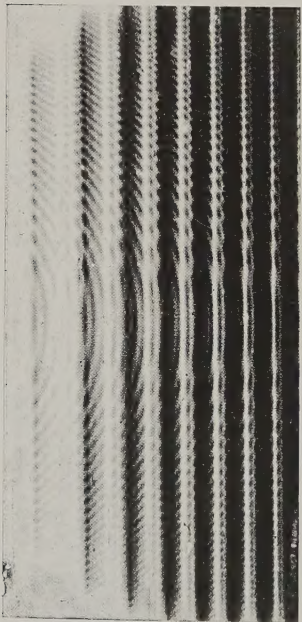
$e=8{\cdot}0$ *cm.*
L.G.+F.P.

Fig. 1c.
λ 5,461.



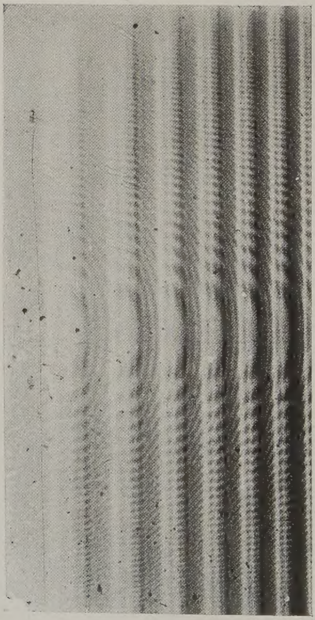
$e=9{\cdot}5$ *cm.*
L.G.+F.P.

Fig. 3a.
λ 4,359.



$e=5{\cdot}0$ *cm.*
L.G.+F.P.

Fig. 3b.
λ 4,359.



$e=6$ *cm.*
L.G.+F.P.

ace p. 97.

In these combinations, let us pass from P to $P+\Delta P$ by keeping h constant, then

$$\Delta x=0$$

and
$$\Delta y=-\frac{\lambda}{2ey}\Delta P=-\frac{1}{P_0 y}\Delta P,$$

giving the change in y. Next vary h and keep P constant; then x and y are to be found from the above two equations by putting $\Delta P=0$; evidently

$$\Delta x=\frac{\lambda^2 h}{2t^2 x}\Delta h=\frac{2(\mu^2-1)}{x}\frac{\Delta h}{h}$$

$$\Delta y=-\frac{\lambda^2 h}{2t^2 y}\Delta h=-\frac{2(\mu^2-1)}{y}\frac{\Delta h}{h}$$

approximately.

These simple relations will help us to understand how the curves of interference points are crowded, as we pass outwards from the centre of the rings, showing that the variations of the co-ordinates for successive spectra are inversely proportional to them.

§ 12. From the above discussion, it will be seen that for calculating the difference in wave-length of interference points referred to the L.G. plate, there is great inconvenience in the gradual decrease in the intervals between the successive spectra as the order h increases from that at grazing exit. Interpolation by simple proportional parts is inadmissible, so that we have to take the second differential into account. This is a rather tedious process, so that for interpolation, the graphical method is perhaps the easiest and the least liable to error. This process was followed in evaluating the readings of the micrometer.

§ 13. Figs. 1 (A, B, C) show the photograms of the line λ 5,461 obtained by the above-mentioned combination for three values of e. The principal line is evidently a triplet with a strong satellite on the positive and negative sides. These points are connected by deep shades, as the resolving power of the plate is not sufficiently high; it is on account of the high resolution of the F.P. interferometer that these points appear distinctly. Several photographs for different values of the air space were taken, all leading to the same conclusion; their distances are as folows :—

−19·9 −8·4 0 +8·2 +18·4 m.Å.U.;

the remaining points are due to satellites, of which the outermost one −237 is conspicuous, and may be used as a reference line for comparing the results of different observers, as suggested by Stansfield. For finding the position of other satellites the air space of about 1 cm. is preferable to that above mentioned. The following table will show how the results can be reconciled with each other, if the lines be referred to the said satellite, and not to the principal line whose position is very vague in ordinary experiment, owing to its multiple character. The last column gives the difference in wave-number per centimetre referred to the principal line :—

Von Baeyer.	Nagaoka & Takamine.		Janicki.	Nagaoka.	
L.G.+L.G.	Echel.+L.G.		L.G.+Wedge	F.P.+L.G.	$d\nu$.
0	0	0	0	0	+0·793
136	133	134	134	135	+0·341
169	166	168	168	167	+0·234
188	191	188	188	190	+0·157
213	220	216	214	216·9	+0·066
...	...	...	227	228·4	+0·027
237 ?	242 ?	246 ?	236	236·8	0
...	...	...	245	245·0	—0·029
...	...	...	254	255·2	—0·062
319	318	320	320	322	—0·285
362	364	365	364	365	—0·431
448	450	452	450	451	—0·717

The above results were all obtained by crossing the two instruments mentioned under the names of the observers ; the comparison shows that the position of the satellites and their number are now well settled for the green line.* The mutual agreement of these numbers affords the proof of the accuracy of the method of crossed spectra. By crossing the echelon grating with L.G. interferometer and utilising the method of coincidences† between the interference points of these satellites, we have measured the distance between the satellites 0 and 365 accurate to four figures and found it to be 365·0. Relying on the echelon spectrum only, which in the present case was better defined than in the former experiments, I have, after repeated micrometric measurements, found it to be 365·0, which exactly coincides with the number obtained by a more accurate but complicated process. It further lends weight to the reliability of echelon spectroscope in analysing the structure of spectrum lines, although the apparatus is more complex and susceptible to changes in the instrumental constants than a L.G. plate, as is evident from the frequent appearance of ghosts in the spectra produced by it.

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* See Note added to the present Paper (p. 114).

† Nagaoka and Takamine, " Phil. Mag., 27, p. 123, 1914.

§ 14. In our former communication, no attempt was made to arrange the satellites according to the difference in wave-number. If they are arranged according to the numbers of the last column in the Table (p. 98), we find that there is a remarkable symmetry in the distribution of the satellites in different lines. In Fig. 2, the first row represents the general aspect of

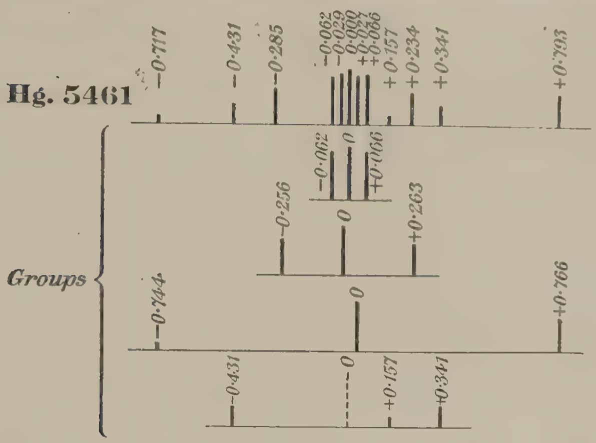


FIG. 2.

the green line, with the intensity taken proportional to the length of the line. Arranging them in groups as indicated in the following four rows, we find a remarkable symmetry in the distribution of the satellites.

The principal line forms a triplet with −0·062 and +0·066 as components; the satellite −0·029 then forms another triplet with the components −0·263 and +0·256 referred to it, being nearly four times the distance between the components of the principal triplet; then +0·027 forms another triplet having the components −0·744 and +0·768 referred to it, the interval being about three times that in the second triplet and 12 times that in the first triplet. The remaining satellites −0·431, +0·157 and +0·341 have apparently no regularity. Further, the components of the triplet have symmetry in the intensity of lines in the first and second groups, but in the third this is no longer the case. The wave-number in the last row is in a singular ratio. The difference in the wave-number intervals counted from the principal of the first positive and negative component $=0{\cdot}274$, and the difference between the first and the second positive components $=0{\cdot}184$; these numbers are in the ratio 3:2.

In addition to this, we find that the interval between the satellites

$$-0{\cdot}717 \text{ and } +0{\cdot}341=1{\cdot}058$$
$$-0{\cdot}285 \text{ and } +0{\cdot}793=1{\cdot}078.$$

The mean of these two numbers $=1{\cdot}068$. It will be afterwards shown that for several mercury lines, the difference in wave-numbers of satellites is constant and about equal to the above number, so that this is not characteristic of the green line only.

§ 15. The complexity of the violet line λ 4,359 is well known, but the multiple structure of the principal line was not yet studied. The arrangement being the same as before, the crossed echelon spectrum obtained by short exposure showed at once that the principal line is complex, but the components are so close together, that 10 m.A.U. were difficult to separate. Crossing the plate with the F.P. interferometer, the complex structure was made out as shown in Figs. 3 (A, B). The interference points are arranged in a manner similar to those in the green line, the principal forming a triplet, of which the middle component is rather weak compared with the side components, each of them being associated with a strong satellite. Perhaps the middle of the triplet, though weak, can be taken as the principal line. The lines of the group are distant by the following amount:—

$$-17{\cdot}9 \qquad -5{\cdot}8 \qquad 0 \qquad +5{\cdot}7 \qquad +16{\cdot}2 \qquad \text{m.Å.U.}$$

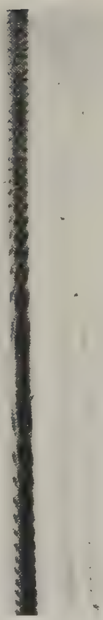
For determining the position of other satellites, I have used crossed echelon spectra. The table given on next page gives the comparison referred to the strong outermost satellite which is easily oriented. The difference of wave-numbers in the last column are not calculated from what appeared to be the principal line, but the initial point is laid at the middle of two strong lines 157·7 and 163·4, so that they form a strong doublet.

§ 16. In our former experiments with an echelon grating and a L.G. plate, the exposure was limited to 20 or at most 30 minutes, but in the present experiment with an echelon and F.P. instrument, the satellites already found were distinctly visible with exposure of 20 minutes; when it was prolonged to 40 minutes, additional lines appeared, which evidently were not to be identified with ghosts; by exposure for two hours,

FIG. 4A.
λ 4,359.

Ech.+L.G.
Exp. 60 min.
(Compare with Fig. 22 of the former communication, Vol. 25 of the "Proceedings.")

FIG. 4B.
λ 4,359.

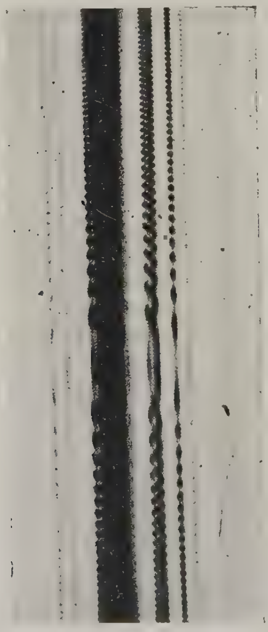


e=5·0 cm.
Ech.+F.P.
Exp. 20 min.

FIG. 4C.
λ 4,359.

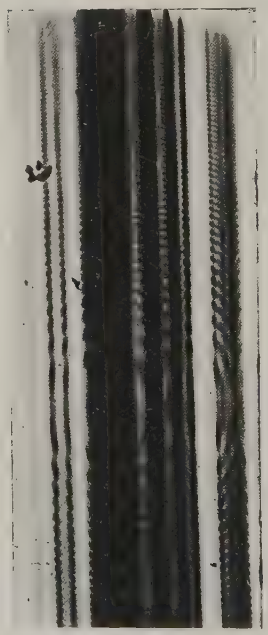
e=10·5 cm.
Ech.+F.P.
Exp. 30 min.

FIG. 4D.
λ 4,359.



e=7·0 cm.
Ech.+F.P
Exp. 40 min.

FIG. 4E.
λ 4,359.



e=8·0 cm.
Ech.+F.P.
Exp. 120 min.

To face p. 101.

Von Baeyer.	Nagaoka and Takamine.		Nagaoka.	
L.G.+L.G.	Echel.+L.G.		L.G.+ or Echel.+F.P.	
...	...	...	−78	+1·254
...	...	...	−57	+1·082
0	0	0	0	+0·844
52	53	53	53	+0·566
66	67	66	65	+0·502
114	116	115	114	+0·245
142	142	140	139·8	+0·110
...	...	...	151·9	+0·047
159 ?	161 ?	163 ?	157·7	+0·015
...	...	...	163·4	−0·015
178	181	180	179·6	−0·099
187	193	189	190	−0·154
203	206	206	207	−0·244
273	267	268	268	−0·566
...	...	...	318	−0·834
344	344	345	343	−0·956
...	...	...	383	−1·176
...	...	...	398	−1·251

the group about the principal was completely darkened, but new lines became distinctly visible. Trying similar experiment with L.G. plate and echelon, I noticed traces of faint lines apparently near the strong lines of the principal group, and though much obscured by them, the course of the interference points indicated that the faint dots belong to echelon spectra different from the principal. The photograms are shown in Figs. 4 (A, B, C, D, E). These satellites are nearly coincident with those found by Wood with a 6 in. concave grating. Although the identification is not perfect, the difference is probably to be ascribed to the error of observation. These are −78, −57, +318, +383 in the above Table.

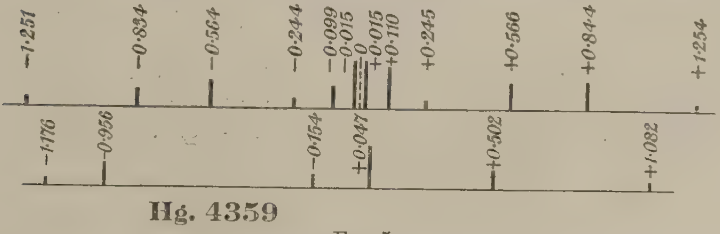


FIG. 5.

§ 17. Plotting the differences of wave-number (Fig. 5), we get two rows of which the upper one shows remarkable symmetry. The intensity of −0·099 is somewhat less than that of +0·110, and for these lines the symmetry is not so exact as for other satellites in the same row. The components near the initial point, *i.e.*, −0·244, −0·099, −0·015 and +0·015, +0·110, +0·245, evidently form a sextet, and the outer components −1·251, −0·834, −0·564 and +0·566, +0·844, +1·254 another sextet, which is more symmetrical than the inner. The wave-number interval of the outer sextet of λ 4,359 is so related that the mean interval of the inner components 0·274 is to the mean of the outer 0·414 in the ratio 2:3. This regularity is indeed remarkable and will no doubt have some intimate connection with the atomic structure. It must not, however, be forgotten that the principal line, which is evidently represented by the triplet −0·015, +0·015 and +0·047 is split into a sextet in magnetic fields, so that the structure here exhibited may have an important relation to the Zeeman effect of the lines. It is also to be noted that the strong satellites −0·099 and +0·110 show the same mode of separation.

§ 18. The lines in the second row have apparently no symmetry, but some of them show striking coincidences in the frequency difference with components of both rows. As to the row of satellites having no apparent symmetry, we can detect the same regularity as for the green line. The difference in absolute wave-number for the first positive and the second negative components =0·909, and the same quantity between the first two positive components =0·455; these numbers are in the ratio 2:1. Further the interval between the principal −0·015 and the second negative component of the same row =1·161, which is nearly double the difference in wave-number of the second and third positive satellites =0·580.

The wave-number interval between

−1·176 and −0·099=1·077
−0·956 ,, +0·110=1·066
−0·834 ,, +0·245=1·079
−0·564 ,, +0·502=1·066
−0·244 ,, +0·844=1·088
+0·015 ,, +1·082=1·067

Mean .... =1·074

A similar interval in λ 5,461 (§ 14) has a mean interval 1·068, which is almost equal to the number found for λ 4,359. Moreover, the interval between

−1·251 and −0·244=1·007
+0·245 ,, +1·254=1·009

Mean ...... =1·008

The difference between these two numbers =0·066, which is the interval between +0·566 and +0·502, and also corresponds to the interval in the triplet found in the green line. These numerical coincidences are apparently not a mere chance, as will be further shown for the line λ 4,047.

§ 19. Experiments similar to those on the lines λ 5,461 and λ 4,359 were made on the violet line λ 4,047. The optical constants of the L.G. plate were such that several satellites came on the same line with the principal, so that even with crossed spectra, the structure was not distinctly observed as with the lines above described. Crossed echelon and F.P. interferometer spectra were more definite, and gave points, which showed that the principal line is a triplet accompanied by a satellite. The wave-number differences counted from the principal are as follows :—

−0·715, −0·362, −0·025, 0, +0·024, +0·075, +0·290, +0·375, +0·708.

It is rather difficult to estimate the resolving power used in the experiment, as the reflection is much enfeebled for such short waves and the absorption in the echelon is much increased, but at the distance of 6 cm. or 7 cm. it will be over $1{\cdot}5\times 10^6$, which is no doubt sufficient to effect the above resolution. Plotting the above number in Fig. 6 we find that there is a triplet at the middle, and two lines on both sides are symmetrically situated about the principal. There are two other lines which are evidently not included in the same group. With this line, the interval in wave-numbers between

−0·715 and +0·375=1·090
−0·362 and +0·708=1·070

Mean ...... =1·080

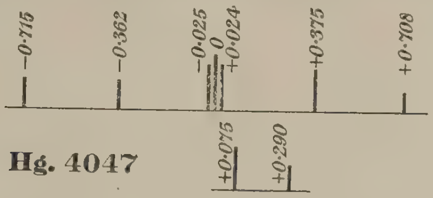


Fig. 6.

The occurrence of this number in λ 5,461 and λ 4,359 was already noticed; moreover, the ratio between the mean components 0·368 and 0·712 to 1·080 is as 1:3 and 2:3 respectively.

§ 20. The similarity in the distribution of the satellites and the occurrence of the same wave-number intervals may be attributable to the character of the second subordinate series to which the three lines here mentioned belong. Arranging the wave-number intervals in another manner, we find that there are many numbers which are common to two of them or to all the three lines. As to the initial position, we shall take +0·793 for λ 5,461, +0·844 for λ 4,359, and +0·708 for λ 4,047. Tabulating the lines we find the following coincidences indicated by dots :—

5,461	4,359	4,047
	+410	
	+238	
0	0	0
	−278	
	−342	−333
−452		−418
−559		
	−599	
−636		−633
		−684
		−708
−727	−734	−733
−766		
−793	−797	
−822	−828	
−855	−859	
	−943	
−1,078	−1,088	−1,070
−1 224		
	−1,408	−1,423
−1,510		
	−1,678	
	−1.800	
	−2,020	
	−2.095	

These numbers are expressed in units 1,000 times smaller than those already used. The table shows that the three lines are not independent of each other, and the satellites are not arbitrarily distributed. It is a question why such coincidences are not found with reference to the principal lines.

Owing to the symmetry in the distribution of the satellites in λ 4,359, the coincidence above stated can be found in the same manner by taking the initial point at −1,678 in the above schedule. Most of the numbers are reproduced, so that by proceeding backwards, we meet with most of the numbers found for λ 5,461 and λ 4,047 already given.

§ 21. Turning our attention to the lines of the other series, we see that in the mercury line λ 5,769 (Fig. 7) there is a triplet about the principal with the wave-number difference −0·139 and +0·147. It is also the case with the line λ 4,078 (Fig. 8), in which the difference amounts to −0·283 and +0·285; this interval is nearly double that between the outer components in the triplet of λ 5,769. We notice that the interval between the two outer negative components of λ 4,078 is 0·146, which is equal to the distance of the first positive component from the principal in λ 5,769. Moreover, the second positive component of the line is distant from the principal by 0·360, which is nearly double the distance of the principal line λ 4,078 from the first negative component.

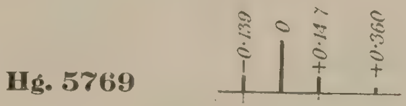


Fig. 7.

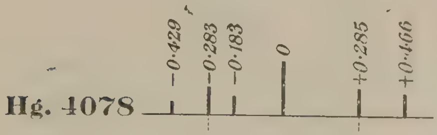


Fig. 8.

Further, we notice that the intervals between the three lines −0·137, +0·147 and +0·360 in λ 5,769 being 0·284 and 0·213 are in the ratio 4:3. Obviously the components of λ 4,078, −0·183, 0, +0·285, +0·466 (=0·285 +0·181) form a quartet. Further experiments on the principal line of 4,078 showed that it consists of three other satellites on the side of lower frequency, at a constant interval of about 0·039, but as these components were difficult to observe with the L.G. plate and F.P. interferometer combination, the position is not indicated in the figure. The structure of the principal line λ 5,769 was difficult to analyse on account of the low sensitivity of photographic plate; some photograms showed that it is a close triplet.

§ 22. The structure of the mercury lines λ 3,663, 3,655, 3,650 belonging to the first subordinate series is shown in Fig. 9, and is taken from the measurements of Mr. Takamine.* Here we notice the same constant difference in the wave-number.

For 3,663, interval between −0·678 and +0·395=1·073
,, 3,655 ,, ,, −0·546 ,, +0·518=1·064
,, 3,650 ,, ,, −0·374 ,, +0·712=1·086

Mean =1·074

This interval was already noticed in connection with the lines belonging to the second subordinate series, and seems to be characteristic of most mercury lines. In the above-mentioned lines there is another interval which is common to them; the difference in wave-number

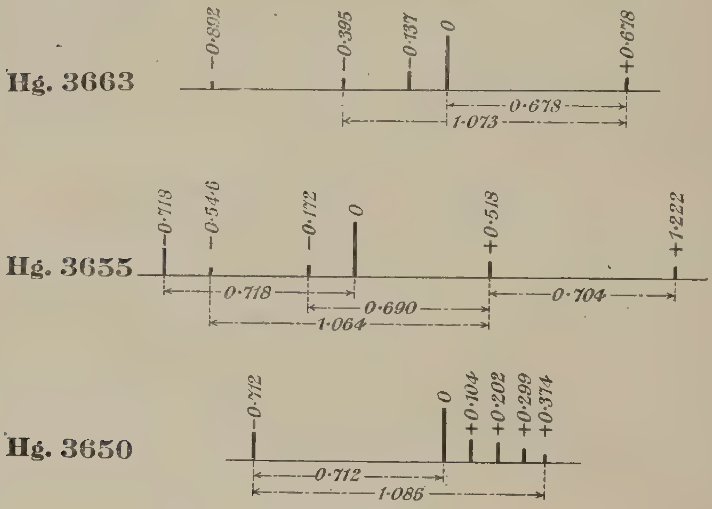


FIG. 9.

For 3,663, between −0·678 and 0=0·678
,, 3,655 ,, −0·718 ,, 0=0·718
,, 3,655 ,, −0·172 ,, +0·518=0·690
,, 3,655 ,, +0·518 ,, +1·222=0·704
,, 3,655 ,, 0 ,, +0·712=0·712

Mean =0·700

The ratio of this number to another constant 1·07 is about 2:3. We meet with a similar number in λ 4,047, in which the outermost components have the mean value 0·711 which stands to 1·07 in the ratio 2 : 3 more accurately than for the three lines just discussed.

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* Takamine, " Proc.," Tokyo Math. Phys. Soc., 8, p. 309, 1915.

It is worth noticing that the intervals given by $l\frac{c}{3}$, $m\frac{c}{4}$, and $n\frac{c}{5}$, where $c=0·71$ and l, m, n are whole numbers, are found scattered among the satellites of the eight mercury lines above mentioned, for $l=1$ to 4, $m=1$ to 7, and $n=1$ to 7.

The second positive satellite of the line 3,663 is nearly three times as distant from the principal as from the first, and the negative satellite is five times the same amount. The first positive satellite of the line 3,655 is distant from the principal by three times as much as the first negative, and the second negative satellite appears as the difference between the first and the third satellite, thus forming a quartet. The difference between the first and the second satellite on the positive side is 0·704 and is nearly equal to the distance of the first positive satellite in 3,650. The components −0·172, +0·518, +1·222 form a triplet.

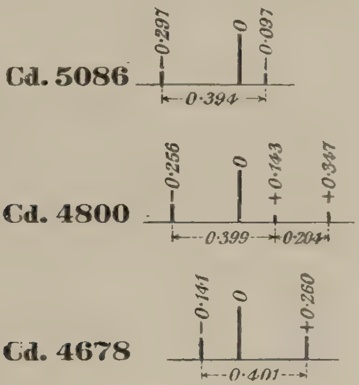

Fig. 10.

§ 23. According to the investigation of Mr. Takamine,* the structure of the cadmium line is simpler, and the regularity more apparent. The lines λ 5,086, λ 4,800, λ 4,678 belong to a triplet series, corresponding to the mercury lines first discussed. These lines have satellites as shown in the difference of wave-number in Fig. 10.

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* Takamine, "Proc." Tokyo Math. Phys. Soc., 8, p. 51, 1915.

As already noticed, we meet with a constant interval in the wave-number of the satellites as follows :—

For 5,086, interval between −0·297 and +0·097=0·394
,, 4,800 ,, ,, −0·256 ,, +0·143=0·399
,, 4,678 ,, ,, −0·144 ,, +0·260=0·401

Mean =0·398

This number evidently corresponds to 1·07 and 0·71 already found for mercury lines. Further, we have to remark that for λ 5,086, the interval between −0·297 and 0 is equal to 3×0·099, while the interval between the principal and the positive satellite is 0·097 ; similarly for λ 4,800 the interval between the negative and the first positive component =0·399 is nearly double the interval 0·204 between the components. With λ 4,678, the distribution of the satellites is reverse to that for 4,800, for in place of −141 and +260 in the former, we find +143 and −256 in the latter. Thus the satellites of these three lines seem to be mutually dependent, as already noticed for the mercury lines This singular fact would have been noticed by Mr. Takamine, if he had reduced his measurements to wave-numbers. It is customary to give the position of the satellites in wave-lengths, but from the theoretical point of view it is advisable to give the wave-number.

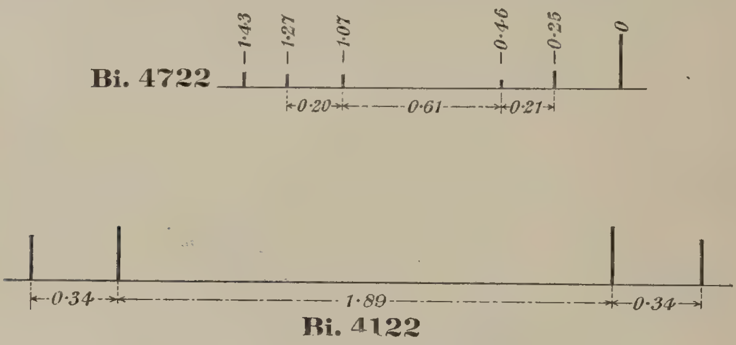

Fig. 11.

§ 24. Similar features are shown by the bismuth lines λ 4,722 and λ 4,122. They are shown in Fig. 11 : the interval between the components −1·27 and −1·07 =0·20 and that between −0·46 and −0·25 =0·21, while that between −1·07 and −0·46 =0·61, being three times the above amount. λ 4,122 consists of a strong doublet at an interval of 1·89, having the satellites at equal intervals of +0·34 and −0·34 from each of them; it is also to be noticed that the interval between the doublets in the latter line is nearly three times that of the former.

§ 25. It would be premature to inquire into the cause of the regularity of the satellites mentioned in the preceding pages, as the data on which the discussion rests are as yet scanty, and the effects of the electric and magnetic fields on the behaviour of spectrum lines are not yet well known. The presence of magnetic and electric fields in atoms is a matter beyond doubt, and changes in interatomic fields will be felt in the vibrations of electrons. It is only recently that Stark has been able to establish definitely the effect of external electric field on emission lines, while that of the magnetic field, although well known for about 20 years, has not yet been satisfactorily explained, excepting that of the normal triplet, which forms only one of the numerous types of separation. If the said effect had been more accurately investigated, it would perhaps not be difficult to found a theory of the existence of the satellites, as the most plausible explanation seems to lie in the change of the quasi-elastic force giving rise to light vibrations, due to the variation of the interatomic fields. The theory of the effect of an electric field on the spectrum line has been discussed by Voigt,* and of the joint action of the electric and magnetic forces by Zeeman.† The investigation of positive rays by Sir J. J. Thomson has revealed the existence of atoms of mercury, which have lost one and sometimes eight electrons at one time. The change in the electric field of mercury atoms by the liberation of so many electrons will have sufficient influence on the radiation by producing an appreciable disturbance of the field, that the wave-length is slightly altered; most probably the change thus caused in the electronic configuration will alter the atomic magnetic field to such a degree, that it will sensibly affect the radiation sent out by the mercury atoms. If the investigation begun by Thomson be extended to different elements, it will shed much light on the nature of the spectrum lines, but in the present stage of our knowledge it is rather difficult to advance a new hypothesis as regards the presence of satellites on the basis of the change of atomic electric and magnetic fields.

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* Voigt, "Ann. der Phys.," 4, p. 197, 1901; "Magneto- u. Elektro-optik," 1908.

† Zeeman, "Akad. v. Wetens.," Amsterdam, May 27, 1911.

§ 26. In his theory of the vibrations of electrons in an isotropic quasi-elastic electric field, Voigt has shown, under the assumption of the existence of potential, which contains terms of higher order than that usually assumed in elasticity, that the change of frequency is proportional to the square of the field; observed transverse to the field, the original line is separated into two components of the same frequency, and parallel to the field there is a displacement three times as large as that of the line vibrating perpendicular to the field. Extending the analogy of the Zeeman effect in an electric field, Voigt has further shown that we shall have a quartet. In the application to mercury atoms it is natural to assume that the result will conform to the case in which the electrons are coupled, because the green line is divided into a nonet in a magnetic field, the violet line λ 4,359 into a sextet, and λ 4,047 into a triplet of double the normal separation, all of which can be explained from the standpoint of the coupled electrons. If we now assume that such coupling takes place in the electric field in the same way as for the magnetic, we can without much difficulty explain the regularity observed in some lines of mercury, if not the whole of it. In other cases, Zeeman's calculation, based on a simple Lorentz's hypothesis as regards the action of magnetic force, lends support to the explanation of the appearance of the satellites in cadmium and bismuth lines.

§ 27. The discussion of the effect of an electric or magnetic field indicates the nature of the polarisation to be observed if the direction of the field be fixed, but in case the seat of such fields lie in the radiating atoms, the polarisation observed becomes promiscuous and cannot be discriminated. With the green line (Fig. 2), taking $-0{\cdot}06$ as the initial point, we notice that the zeros of the lines as arranged in the third and fourth row in the figure are distant from it by 0·03 and 0·09, being in the ratio 1:3 as expected on Voigt's theory. Starting from these lines, we have the lines $\pm 0{\cdot}26$ in the third row and $\pm 0{\cdot}75$ in the fourth, thus affording a solution in the light of coupled electrons. The last row consists of an unsymmetrical triplet, which can be caused by the joint action of the electric and magnetic forces as indicated by Zeeman. Another suggestion that we can make of these three symmetrical triplet rows in the diagram is that they are all due to the Zeeman effect, the magnetic field producing these separations being caused by the disturbance in the configuration of electrons in the mercury atom on the expulsion of electrons and their subsequent return to it. But it seems to me that the former way of seeking the solution of the problem is more plausible than the latter; for there are different instances which favour the view that the satellites are due mostly to electric fields. The feeble satellites $-0{\cdot}137$ and $-0{\cdot}395$ in λ 3,663, $+0{\cdot}172$ and $+0{\cdot}546$ in λ 3,655, and $+0{\cdot}104$ and $+0{\cdot}299$ in λ 3,650 are all in the ratio 1:3 and conform to Voigt's simple theory. This is, however, only one aspect of the phenomena; as, for the line λ 3,655, we may consider the group of lines $-0{\cdot}718$, $-0{\cdot}546$, $-0{\cdot}172$, and 0 as forming a quartet having a constant interval of $0{\cdot}172$ at both ends; moreover, the lines $-0{\cdot}172$, $+0{\cdot}518$ and $+1{\cdot}222$ may be looked upon as a triplet with a mean interval of $0{\cdot}697$; as to the line λ 3,650, the said lines may form a group of gradually evanescent lines; thus various arrangements of lines may be considered as possible, for which different solutions can be suggested. If the said lines in λ 3,655 be considered as arranged in a quartet, then they may be due to the action of the electric and magnetic forces which are at right angles to each other. The satellites belonging to the bismuth lines λ 4,722 and λ 4,122 belong to the same type, as indicated in Fig. 11.

As to the line λ 4,047 of mercury (Fig. 6), the middle line may represent the position in the undisturbed state, while the other four lines may correspond to the state in which the coupled electrons vibrate in the electric field, or may be subject to the action of electric and magnetic forces. Near by the principal are two lines $+0{\cdot}024$ and $+0{\cdot}075$ in the ratio 1:3; they thus seem to belong to the lines conformable to Voigt's theory, but the four lines in the upper row are symmetrical about the principal and not about these two; whence it seems plausible that they are produced in the uncoupled condition. The presence of two symmetrical sextets in λ 4,359 (Fig. 5) is not easy to explain; on Zeeman's theory the presence of a dissymmetrical triplet is made possible, and when the presence of the line in zero field is considered as possible, there results a symmetrical quartet when the magnetic force is perpendicular to the electric. If the theory of the Zeeman effect be perfected, so that it can explain different modes of separation, such as a quartet, sextet or nonet, we shall be in a position to explain the distribution of the satellites in λ 4,359. If we exclude the outermost components, which are very faint, and not observed by most experimenters, we can consider the lines $-0{\cdot}834$ $-0{\cdot}564$ and $+0{\cdot}566$, $+0{\cdot}844$ as forming a symmetrical quartet, and $-0{\cdot}244$, $-0{\cdot}099$, and $+0{\cdot}110$, $+0{\cdot}245$ as forming another quartet; these may be supposed as due to the vibrations of coupled electrons in two different electric fields, which may arise during the liberation of electrons from the radiating atom or return to it. Similarly, if the outer satellites in the second row be not taken into account, the lines form a dissymmetrical triplet and may be due to the electric and magnetic forces. Good instances of a dissymmetrical triplet are found in the cadmium lines λ 5,086 and λ 4,678, while the mercury line λ 5,769 and the cadmium line λ 4,800 form dissymmetrical quartets; singular enough is the fact that in the former the intervals among the satellites are in the ratio 3:4 and in the latter 1:2; this makes us consider the cause of these lines as due to some action or arrangement of electrons, which give rise to the above mentioned regularity.

I cannot as yet lay much importance on the theories above given; they may be looked upon as having some significance in the theoretical aspect of the problem.

§ 28. When we consider the different causes which introduce the regularities above mentioned, we naturally fall back on the supposition that the primary cause is of electromagnetic origin. As stated before, the researches of Thomson seem to give some hints to the origin of such forces. As regards the mercury atom we know that it gives out a single electron or eight electrons at once; the former is the usual mode of ionisation, and the emission of the principal lines will also be the consequence of the expulsion of the electrons giving rise to electric and magnetic disturbances. As to the latter way of ionisation, it is peculiar to mercury, and the disturbance arising from it will be of such an amount that the field in which the atomic constituents are placed will be subject to changes, by which the electrons emit intense electromagnetic waves. How often such expulsions of electrons take place compared with the number of expulsions of a single electron is not known, but most probably it will be less frequent, and though the emission may be intense, the total effect will not be so great as for the ordinary single electron emission. Owing to the violent change consequent upon the expulsion of several electrons, the electric field of the atom will be influenced by it, and the vibration of electrons modified to such a degree as to give rise to satellites. It will be at the moment of disruption that the electrons are subject to the most violent acceleration, and are in a state of giving out the most intense radiation. It therefore seems to me probable that the principal lines are due to the consequence of the expulsion of an electron from the atom, and the satellites are due to the disruption of a ring of electrons. It may, however, be argued that on the gradual return of the electrons to the atom, there will be a new adjustment of the electron configuration in the atom, which calls forth radiation; this will inevitably be the case, but the disturbance caused by the return of the electrons to the atom will generally be small as compared with the expulsion of eight at one time, so that the result is not so great as to affect the photographic plate.

§ 29. The frequent occurrence of the common difference in the wave-number for lines of different wave-length calls for a new explanation. According to Bohr, the radiation emitted during a transition between two stationary states of an electron system in an atom is determined by

$$h\nu = A_1 - A_2,$$

where h is Planck's constant, ν the frequency, and A_1 and A_2 are the energies of the system in two stationary states. For a satellite of frequency $\nu+\delta\nu$, a similar equation must hold, so that we shall have

$$h\delta\nu = \delta A.$$

The above fact shows that for different values of ν, $\delta\nu$ has sometimes constant values, and consequently δA is common to these satellites. In other words, the energy difference between two stationary states which give out radiation corresponding to the satellites is constant. Assuming that the change in the stationary states of the emitting system is the effect of electric and magnetic fields, the energy possessed by the system in these states will be altered in such a way that the amount is still some multiple of a certain quantum, which is different from h.

On this hypothesis, the occurrence of such parts in the difference between the wave-numbers of satellites is a natural consequence. This is well exemplified in most of the lines discussed in the preceding pages; the numbers 1·07 and 0·70 found in the mercury lines, and 0·40 in the cadmium lines are remarkable instances of the existence of the quantity just discussed.

Judging from the fact that such common differences are characteristic of the lines belonging to the same series, it would appear that the lines belonging to the same series emit multiples of quanta, which are peculiar to them. If we suppose that the electrons are arranged in rings about a central nucleus, and that they revolve in circular orbits, the above conclusion indicates that the stationary orbits do not vary continuously, but by jumps. This is necessarily due to the quantum change in energy, which will be emitted in the case of mercury atom, when an electron is liberated from it, or when eight electrons are expelled, or on subsequent return of electrons till the atom becomes ultimately electrically neutral. These considerations lead to the conclusion that the existence of numerous satellites in mercury lines is not at all singular, but is a natural consequence of the mode of ionisation peculiar to the element. If the experiment on the positive rays be extended to various elements and its consequence examined in the light of the theory here sketched, we shall perhaps be able to decide the question as regards the origin of the satellites. To account fully for the existence of the satellites and the regularity of their distribution, we are still in want of sufficiently accurate data as to the position of the satellites in order to test the consequences of the hypothesis which has been here advanced.

Note on the Structure of the Green Line of Mercury and the Terms of Correction in Using a Concave Grating.

The structure of the green line of mercury as revealed by Michelson's grating* ($9\frac{3}{8}$ in. × $2\frac{7}{8}$ in.) of 20 ft. focal length in the sixth order spectrum presents some points of discrepancy between that obtained by means of interferometers, echelon gratings, Lummer-Gehrcke plate, or Fabry-Perot apparatus. The principal difference is the obliteration of the strong satellites $-19{\cdot}9$ and $+18{\cdot}4$m.Å.U. from the principal line, which are fused into one line, although the resolving power of the grating would have been sufficient to separate them; the triplet structure of the principal would, however, have been difficult to resolve. This, I believe, is due to terms of correction, which, for gratings of such size account for the overlapping of the neighbouring lines.

Take the x-axis in the direction of the normal to the grating at its middle point, and y, z axes in the tangent plane at the point of the sphere, on which the lines parallel to the z-axis are ruled. Referred to these axes, let the co-ordinates of the source of light A be a, b, o; of the point A' at which a spectrum of hth order is formed a', b', o; and of a point P on the grating x, y, z. Then $AP^2 = (a^2+b^2) - 2\ by\ -2ax + x^2 + y^2 + z^2$;

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* Michelson, "Proc." American Phil. Soc., 54, p. 137, 1915.

putting $r^2 = a^2 + b^2$, and introducing the condition that P lies on a sphere of radius ρ, we get

$$AP^2 = r^2 - 2by + \left(1 - \frac{a}{\rho}\right)x^2 + \left(1 - \frac{a}{\rho}\right)y^2 + \left(1 - \frac{a}{\rho}\right)z^2,$$

and $$A'P^2 = r'^2 - 2b'y + \left(1 - \frac{a}{\rho}\right)x^2 + \left(1 - \frac{a'}{\rho}\right)y^2 + \left(1 - \frac{a'}{\rho}\right)z^2.$$

In the present problem, y may be considered as a small quantity of the first order, and x as of the second order, while in practice z is intermediate between x and y. Expanding in powers of y, we obtain

$$AP = r - \frac{b}{r}y - \frac{a}{2r}\left(\frac{a}{r^2} - \frac{1}{\rho}\right)y^2 - \frac{ab}{r^2}\left(\frac{a}{r^2} - \frac{1}{\rho}\right)y^3 + \dots$$

with similar expression for $A'P$. If we make the terms containing y^2 and y^3 evanescent in these two expressions, we arrive at the usual Rowland's arrangement $r^2 = a\rho$ and $r'^2 = a'\rho$, by which the points A and A' lie on a circle of half the radius of the sphere. When this condition is fulfilled,

$$1 - \frac{a}{\rho} = 1 - \frac{a^2}{r^2} = \frac{b^2}{r^2}.$$

and $$AP^2 = r^2\left\{\left(1 - \frac{b}{r}y\right)^2 + \frac{b^2}{r^4}(x^2 + z^2)\right\},$$

which, on expansion, gives

$$AP = r - \frac{b}{r}\left(1 - \frac{b^2}{2r^4}z^2\right)y + \frac{1}{2}\frac{b^2}{r^3}(x^2 + z^2).$$

Putting the approximate value $x = \frac{ay^2}{2r^2}$, and denoting the angle of incidence by θ, and the angle of diffraction by θ',

$$AP = r - \sin\theta\left\{1 - \frac{\tan^2\theta}{2}\left(\frac{z}{\rho}\right)^2\right\}y + \frac{\tan\theta\sin\theta}{2}\frac{z^2}{\rho} + \frac{1}{8}\tan\theta\sin\theta\frac{y^4}{\rho^3}$$

$+ \dots$ with similar expression for PA'.

Confining our attention to a constant value of z, we get the equation of finite difference with respect to y.

$$\Delta(AP + PA') = \left\{-(\sin\theta + \sin\theta') + \left(\frac{\tan^2\theta\sin\theta}{2} + \frac{\tan^2\theta'\sin\theta'}{2}\right)\left(\frac{z}{\rho}\right)^3 + \left(\frac{\tan\theta\sin\theta}{2} + \frac{\tan\theta'\sin\theta'}{2}\right)\left(\frac{y}{\rho}\right)^3\right\}\Delta y.$$

Let the successive intervals of the ruling along the chord be $\varepsilon=\Delta y$; then, for a spectrum of h's order, we shall have

$$\varepsilon\left\{(\sin\theta+\sin\theta')-\left(\frac{\tan^2\theta\sin\theta}{2}+\frac{\tan^2\theta'\sin\theta'}{2}\right)\left(\frac{z}{\rho}\right)^2\right.$$
$$\left.-\left(\frac{\tan\theta\sin\theta}{2}+\frac{\tan\theta'\sin\theta'}{2}\right)\left(\frac{z}{\rho}\right)^3\right\}=\pm h\lambda.$$

According to the usual way of treating a grating spectrum, the corrections in the second and third terms are not taken into account, so long as they do not exceed $\pm\frac{\lambda}{4}$. If the line be not associated with satellites the assumption would be admissible, but in case the structure of the line is complex, the effect of these terms is to introduce admixture of different lines, and the feeble satellites are liable to be effaced; with strong satellites, the demarcation between the lines becomes indistinct, so that they are amalgamated into a wide line with vague boundaries.

Let λ_0 be the wave-length for $y=0$ $z=0$; then corresponding to the same angles, and for y and z, $\lambda=\lambda_0+\delta\lambda$, where

$$\delta\lambda=\pm\frac{\varepsilon}{2h}\left\{(\tan^2\theta\sin\theta+\tan^2\theta'\sin\theta')\left(\frac{z}{\rho}\right)^2\right.$$
$$\left.+(\tan\theta\sin\theta+\tan\theta'\sin\theta')\left(\frac{y}{\rho}\right)^3\right\}.$$

The deviation from λ_0 increases very rapidly, as the portion of the grating at some distance from the middle is utilised, especially when the angle θ increases and the ruled space amounts to a few hundredth part of the focal length of the grating. The following tables, giving the corrections, at once show the importance of taking these terms into account.

Table of $\frac{\sin\theta\tan\theta}{2}\left(\frac{z}{\rho}\right)^2\times10^7$.

θ $\frac{z}{\rho}=$	1/500	1/400	1/300	1/250	1/200	1/150
10°	0	0	0	0	1	1
20°	1	1	3	4	6	10
30°	3	5	9	13	21	37
40°	9	14	25	36	57	101
50°	22	34	60	81	136	241
60°	52	81	144	208	324	577
70°	142	221	394	567	885	1,576
80°	634	991	1,760	2,534	3,977	7,039

Table of $\frac{\sin\theta\tan\theta}{2}\left(\frac{y}{\rho}\right)^3\times 10^7$.

θ $\frac{y}{\rho}=$	0·01	0·02	0·03	0·04	0·05
10° …………	0	1	4	10	19
20° …………	1	5	17	40	78
30° …………	1	12	39	92	180
40° …………	3	22	73	173	336
50° …………	5	37	123	292	570
60° …………	8	60	202	480	937
70° …………	13	103	348	830	1,615
80° …………	28	223	754	1,786	3,491

These tables show at once how the corrections arising from large values of y and z increase very steeply as the angle of incidence is increased. In addition to this, it must not be omitted that the presence of the term $\left(\frac{\tan\theta\sin\theta}{2}+\frac{\tan\theta'\sin\theta'}{2}\right)\frac{z^2}{\rho}$ in the expression for $AP+PA'$ gives rise to the further intermixture of lines, if in practice we gather the light coming from different points of the ruled spacing.

An example may be cited for a grating of 20 in. focal length with 11,700 lines to the inch; for the green line of mercury λ 5,461 in the sixth order spectrum, a simple calculation shows that $\theta=\theta'=48{\cdot}2^\circ$, and for $y=5$ in., $\delta\lambda=24$m.Å.U., which is greater than the distance of the satellites lying close to the principal line; for $y=10$ in., $\delta\lambda=213$m.Å.U., which greatly surpasses the limit of error usually admitted in wave-length measurements.

It is therefore not at all surprising to find that in the photogram of the green line of mercury obtained with Michelson's 10 in. grating, the principal line is amalgamated with its neighbouring satellites into a single broad line, which far surpasses the other lines in intensity. The ambiguity in the position of the satellites arising from the overlapping of spectra of different orders usually observed with interferometers of high resolving power, and which can be discriminated by the method of crossed spectra, is entirely got rid of in spectra obtained with a concave grating. This is a great merit of the grating with large ruled spacing, and physicists are under great obligations to Michelson that he has brought his instrument to such perfection that no ghost can be detected, and the ruled space is so large that most of the satellites can be determined with accuracy and without ambiguity. What I want to indicate in this note is that there are terms of correction, which make the resolving power of the instrument less than that calculated by the usual formula, and that large values of the angle of incidence and of diffraction in such a grating lead to considerable deviations.

These considerations lead to the conclusion that for the exact delineation of fine structure of lines, a concave grating with a large ruled spacing is not so effective as interferometers of high resolving power, due to the gradual intermixture of neighbouring lines. To remedy this defect, the only course that may be followed is to increase the radius of the sphere on which the lines are ruled. Another means would be to rule at smaller intervals, and thus have such a large number of lines within a narrow space as would make the terms of correction above discussed less disturbing in the result.

ABSTRACT.

The Paper describes a further development of the work done by the author and Mr. Takamine on the distribution of the satellites of the mercury lines. The apparatus employed was a Fabry-Perot interferometer crossed with an echelon or Lummer-Gehrcke plate. An account of the theory governing the interpretation of crossed spectra is given.

It is shown that much of the discord between the results of various observers of these satellites is due to the unsatisfactory nature of the principal line as a datum from which to define their positions, and that if the distances be measured from one of the distinct satellites, good agreement is obtained.

If these separations be expressed as differences of wave-number, instead of wave-length, a remarkable symmetry in their distribution becomes apparent. For example, among the satellites of the green line, 5,461, can be found three groups of symmetrical triplets, of which the wave-number differences are in the simple ratios 1 : 3 : 12.

Similar results are obtained for other lines, the principal component of λ4,359 being shown for the first time to consist of a triplet, of which the middle component is relatively weak.

A similarity in the distribution of the satellites exists for all the lines examined, and certain wave-number intervals are common to all.

It is observed that the intervals given by $l\frac{c}{3}$, $m\frac{c}{4}$, and $n\frac{c}{5}$, where $c=0{\cdot}71$, and l, m, and n are whole numbers, are found among the satellites of the eight lines quoted for $l=1$ to 4, $m=1$ to 7, and $n=1$ to 7.

The possible origin of satellites is discussed in the light of the Zeeman and Stark effects, the magnetic or electric fields being electronic in origin, and many of the results are shown to be consistent with Voigt's theory of the vibrations of electrons in an isotropic, quasi elastic, electric field. The suggested explanations can only be tentative as yet, pending further development of our knowledge of the theory

of the Zeeman effect, and the extension of Sir J. J. Thomson's work on positive rays to a larger number of substances.

The second part of the Paper consists of a mathematical investigation of terms of correction in the theory of large sized concave gratings, which prevent the theoretical resolving power from being realised in practice.

DISCUSSION.

Dr. H. S. Allen thought the Paper contained many interesting and important points. That simple numerical relationships had been shown to hold for the separations of the satellites was likely to prove very helpful to those who were trying to solve the constitution of the atom. He was interested in the discussion of the part which might be played by the magnetic fields of atomic origin in these phenomena. He had himself held this idea for some time.

Dr. R. S. Willows did not quite understand the statement made at the beginning of the Paper that there was, as yet, no observations of the effect of an electric field on the satellites, in view of the experiments of Stark, Lo Surdo and others.

Prof. Nicholson, in reply, said he presumed the statement was intended to refer to the mercury satellites only.

VII. *A Clock of Precision.* *By* C. O. BARTRUM, *B.Sc.*

RECEIVED DECEMBER 6, 1916.

I. INTRODUCTION.

II. DESCRIPTION.

1. The Master Pendulum.
2. Impulse Pallet.
3. Gravity Impulse Lever.
4. Releasing Electromagnet. Circuit I.
5. Restoring Lever.
6. Restoring Electromagnet. Circuit II.
7. The Slave Clock.
8. The Counting Mechanism.
9. The Rating Spring.
10. The Supplementary Rating Spring.
11. The Synchronising Mechanism.
12. The Principle.
13. False Synchronism overcome by negative backlash.
14. Description of Synchronising Mechanism.
15. Mode of working.

III. MATHEMATICAL DISCUSSION.

Table of Symbols.

16. The Figure of the Impulse Pallet. Energy conveyed with each Minute Impulse.
17. The Energy Required to Maintain the Master Pendulum.
18. The Couple Retarding the Pendulum due to Friction of Impulse Wheel.
19. The Rating Spring.
20. The Method of Synchronising.
21. The Timing Wheel.

IV. DISCUSSION OF CERTAIN CAUSES OF VARIATION AND THEIR PROBABLE EFFECTS ON THE RATE.

22. Possible Changes of Condition.
23. Circular Error.
24. Escapement Error.
25. Application to Wheel Friction.
26. Application to Impulse.
27. Change of Value of Friction of Impulse Wheel.
28. Change of Instant of Release of Impulse Lever.
29. Shift of the Suspension Sideways.
30. Change of Conditions of Impulse Lever.
31. Change of Effect on the Pendulum of the Fall of the Wheel.
32. The Best Amplitude.

V. CONCLUSION.

I. INTRODUCTION.

In designing a pendulum clock provision has to be made for automatically counting the swings and for maintaining the arc of the pendulum. In general practice these are effected by some form of release, actuated by the pendulum itself, or of contact with the pendulum whereby the vibrations are recorded and the signal given to the motive mechanism that the pendulum is in the position to receive its impulse. Such methods are unobjectionable so far as the disturbance thereby caused to the free swing of the pendulum is constant. In practice this is impossible.

In the clock described below, with a view to this mechanical disturbance, and the variations of this disturbance, being reduced to a minimum, a division of labour is introduced, an auxiliary or slave clock recording the time and releasing the impulse while a master pendulum controls the slave clock.

The master pendulum, rigidly suspended, is intended to be surrounded with an air-tight case and placed below ground or in a locality as far as possible free from vibration and from change of temperature. The slave clock, connected with the master pendulum with electric wires, can be placed in the laboratory or observatory with only ordinary precautions against disturbing vibrations and variations of temperature.

When the writer designed this clock he was unaware that the method of relieving the regulating pendulum by throwing some of its labours upon a slave clock had been previously applied. He has since learned that the clock built for the Royal Observatory, Cape Town, during the régime of Sir David Gill was on this principle. This latter clock, however, was not a practical success, owing to the uncertainty of a delicate electric contact, and was discarded. No description of the Cape clock has been published, but the account given the writer by Sir David Gill showed it to differ in its application of the slave clock method from the clock now being described.

II. DESCRIPTION.

1. *The Master Pendulum.*—The controlling pendulum vibrates seconds, and is compensated for temperature, the compensation being adjusted for conditions where the surrounding air does not change in density with temperature.*

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* The master pendulum in the model is by Riefler, of Munich, with nickel steel rod of low coefficient of expansion and brass bob. No details are discussed of the method of maintaining the constancy of air-density in the pendulum case, as they are not essential to the subject of the Paper.

2. *Impulse Pallet.*—At its lower end is attached an impulse pallet, preferably of stone,* A (Fig. 1), the upper surface of which is horizontal, passing at one end to a slope as shown in the diagram. The figure of the pallet in vertical section is described in § 16.

3. *Gravity Impulse Lever.*†—A gravity impulse lever BC (Fig. 1), pivoted at C, having a small wheel, D, mounted with jewelled pivots at its distal end, is held in a nearly horizontal position by a detent, E. The lever is adjusted by the screw F, so that the periphery of the wheel is as near as possible to the surface of the pallet without danger of its touching.

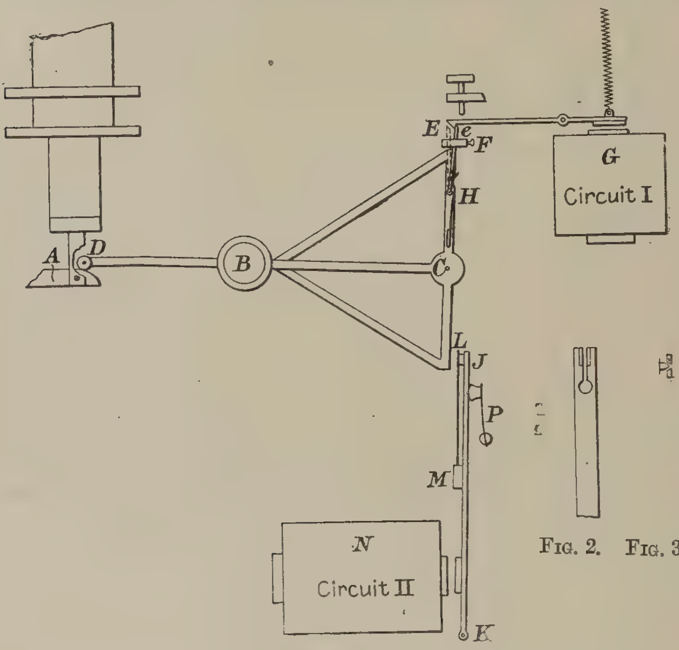

Fig. 2. Fig. 3.

Fig. 1.

The Lower End of the Master Pendulum and its Impulse Mechanism.

4. *Releasing Electromagnet, Circuit I.*—Once in every minute, when the pendulum is approaching its zero position

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* The impulse pallet in the model is of steel.

† The general form of the gravity lever adopted was probably suggested to the writer's mind by the detached gravity escapement of Sir H. H. Cunynghame.

towards the left, an electric current, I, from the slave clock, energising the electromagnet G, releases the impulse lever, the wheel drops through a fraction of a millimetre on to the pallet, rolls down the impulse slope and falls free, giving the pendulum sufficient energy to maintain its swing at the required amplitude.* The period during which the impulse lever is engaged with the pallet is a small fraction of a second,† and for the remainder of the minute the pendulum is free. The catch mechanism E is so arranged that the lever is free from the moment of its release, the pawl, *e*, being driven out of contact with the detent by the spring H. The only forces acting on the impulse lever during its fall are gravity and the reactions at C and D.

5. *Restoring Lever.*—The impulse lever after leaving the pallet falls freely through a short distance, and then strikes a restoring lever, J K. The latter carries on the face that is struck a light spring, L M, electrically insulated at M from the body of the lever, and bearing at its end, L, a wedge-shaped platinum contact piece, L. The restoring lever is split at its end, J (*see* Figs. 2 and 3), the edges of the split being faced with platinum.

6. *Restoring Electromagnet, Circuit II.*—The impulse lever on falling upon the restoring lever drives the platinum surfaces into good contact, thereby closing an electric circuit, II, which, through the medium of the electromagnet N, throws the impulse lever up into its resting position, where it is retained by the detent.

The stop against which the restoring lever rests consists of a spring, P, of such elasticity that the period of electric contact is sufficient for the electromagnet to become saturated before the lever is thrown back. This insures an absence of "chattering" and premature break of the current.

7. *The Slave Clock.*—The slave clock is an ordinary seconds regulator. It is maintained in synchrony with the master pendulum by a method described in (§§ 11 to 15).

8. *The Counting Mechanism.*‡—As mentioned above (§ 4), at intervals of one minute an electric current controlled by this clock releases the impulse lever of the master pendulum. This

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* The semi-amplitude adopted in the model is 1 deg. (*see* § 32).

† In the case of the model this period is about 0·18 second.

‡ The writer owes the idea of a counting wheel to Mr. Hope Jones.

is effected as follows: In the Fig. 4, A is a counting wheel of 30 teeth, B D is a short pendulum with a gathering pawl, C, at its lower end. By means of a horizontal connecting rod, D E, the pendulum moves the wheel one tooth every complete vibration. On the face of the wheel is a platinum pin, F, which at one position of the wheel passes between two platinum contact points, G, closing the electric circuit I. The length of the connecting rod is so adjusted that this momentary closing

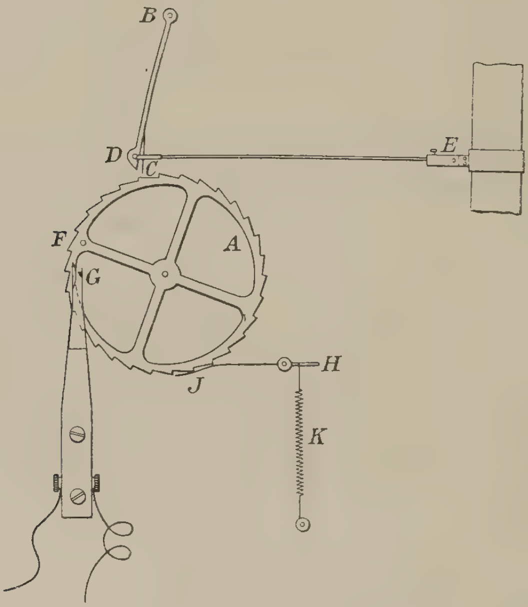

Fig. 4.—The Counting Mechanism.

of the electric circuit is effected when the slave pendulum is at its zero position. The length of the short pendulum B D bears such a relation to the radius of the wheel, and the normal amplitude of the point of the main pendulum which drives it, that the gathering pawl is free of the wheel except near the middle of its path. A light lever, H J, ending in a thin flat spring, J, presses on the teeth of the wheel and also functions as a ratchet pawl. The pressure of this spring can be adjusted by altering the tension of the spiral spring K. It is important that this counting mechanism be as light and as little absorbent of energy as possible, in order not to overburden the pendulum and the train of the clock.*

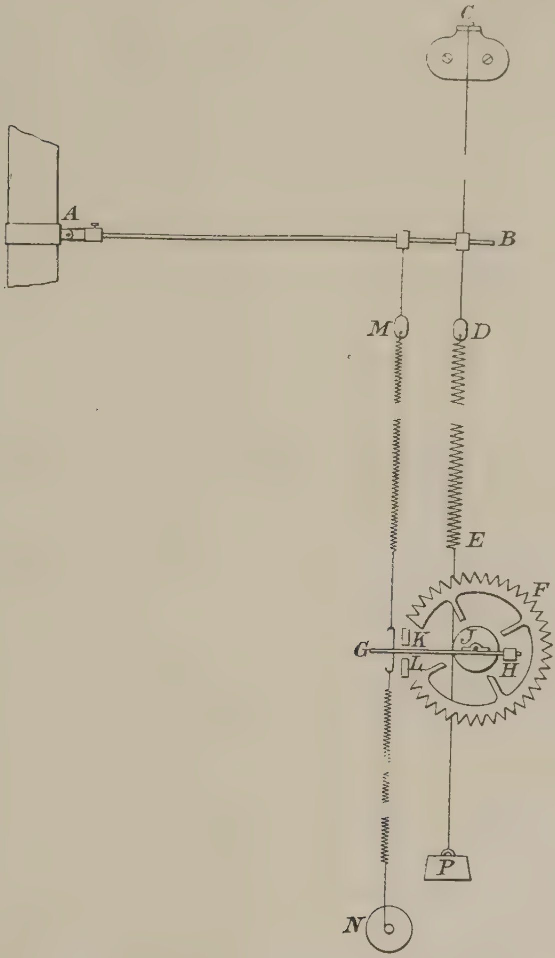

Fig. 5.—The Principal and Supplementary Rating Springs.

9. *The Rating Spring.*—On the opposite side of the pendulum to the counting mechanism is hinged a light horizontal rod, A B (Fig. 5), its distal end suspended by a fine wire to a

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* A light contact on the 'scape wheel, if adopted in place of this counting wheel, would absorb less energy, but it is doubtful whether such a contact would define the epoch with such a degree of accuracy. Such accuracy is essential for the close synchronising of the two pendulums.

cock, C, on a level with the cock of the pendulum. Below the horizontal rod, and attached at the same point as the suspension wire is a spiral spring, D E. The lower end of the spiral spring is continued by a watch fusee chain, which is wound round a barrel on the arbor of the star-wheel F. When the pendulum is at its vertical position the rod A B is horizontal and the suspension wire and spiral spring are vertical. An increase of the tension of the spiral spring brought about by turning the star-wheel F will increase the rate of the clock (§ 19); moreover, this mode of altering the clock's rate is free from appreciable backlash,* a point that will be found later (§ 13) to be of importance. A small weight, P, about equal to the tension of the spring at its mean value, hangs by a silk thread wound round the same barrel, and renders the star-wheel subject only to the small difference of the forces of the two.

10. *Supplementary Rating Spring.*—A rocking lever, G H, friction-tight on the arbor J of the star-wheel, is limited in its movements by the two stops K and L; a supplementary spiral spring, M G, parallel with the above, is attached by its upper end to the horizontal rod at M and by its lower end to the rocking lever at G. This second spring is counterpoised by an exactly similar spring attached by its upper end to the rocking lever and by its lower to a friction-tight arbor, N. This arbor can be turned by a milled head for the adjustment to equality of the tension of the two springs M G and G N, when the lever remains indifferently against either stop. The function of this supplementary spring will be explained later (§ 13).

11. *The Synchronising Mechanism.*—In §§ 4 and 6 two electric circuits were referred to—viz., (I.) that closed by the counting wheel, whereby the impulse lever of the master pendulum was released, and (II.) that closed by this lever on its fall, and whereby it was restored to its initial position. These two circuits pass also through electromagnets in the synchronising mechanism.

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* If, when the star-wheel, moving clockwise, reaches a certain position, the tension of the spring is the same as that attained when the star-wheel, moving counter-clockwise, reaches the same position, there is no backlash. In general, with such arrangements, the tension is less while the motion is in the direction to increase the tension. In the above arrangement the difference is very small.

12. *The Principle.*—The principle of the method of synchronising the slave clock with the master pendulum is as follows: Every minute the impulse lever is released at an instant dependent on the position of the slave pendulum. The lever rolls on the pallet and falls on its contact at a time dependent on the position of the master pendulum. There is an interval, t, of about $\frac{1}{4}$ second between the closing of circuit I. and circuit II. It is the function of the synchronising mechanism to maintain this interval constant. When the interval is greater than t the slave pendulum is in advance and its error is positive. Every minute a measurement of this interval is automatically made, when, if the error of the slave pendulum is positive, its rate is reduced by a constant amount, N; if negative, the rate is increased by the same quantity. These changes of rate are effected by altering the length of the rating spring (§ 9), and, therefore, its tension, by a constant quantity.

13. *False Synchronism.*—It is shown in § 20 that this method alone will result in a false synchronism, a hunting of error and rate, the error and rate passing through periodic variations of constant amplitude, the variation of the error following that of the rate with a difference of phase of a quarter of a period. Further, it is there shown that if there is any backlash of the rating mechanism, in the sense in which that word is used in § 9 Footnote, the amplitude of these periodic variations of error and rate will continually increase without limit. The function of the supplementary spring is to introduce a negative backlash, so to speak, which effectually reduces the amplitudes of these variations till they nearly disappear. When, by any change of conditions, the rate of the slave clock is altered, periodic variations of the error and rate are set up, which in course of time are damped out, and the rate of the slave pendulum readjusted to that of the master pendulum. In practice it is found that there is a continual wavering, of small amplitude, of the error and rate about a mean position,* this mean position continually adjusting itself to changing conditions.

14. *Description of Synchronising Mechanism.*—The electric current of circuit II. passes through the electromagnet A (Fig. 6). The armature B of this magnet is attached to the lever C (Fig. 6 and 7), which is held away from the poles of the

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* With the experimental model the amplitude of these variations of error rarely exceed ±0·005 (*see* Footnote to § 20).

magnet by the spring D. Attached to the end of this lever is the tongue E (Figs. 6, 7 and 8), movable about a vertical axis through a small distance of about twice its horizontal thickness determined by two stops (not seen in the figures) and sliding with a little friction on its guide. The end of this tongue is shown enlarged in Fig. 8. Engaging with the star-wheel referred to in § 9, Figs. 5 and 7, are two pawls, G G (Figs. 7 and 8), movable about pivots, H H, and held against the stops J J by the springs K K. The points of these pawls when pressed upon the wheel F move it to the right or left, the movement through the space of one tooth being completed

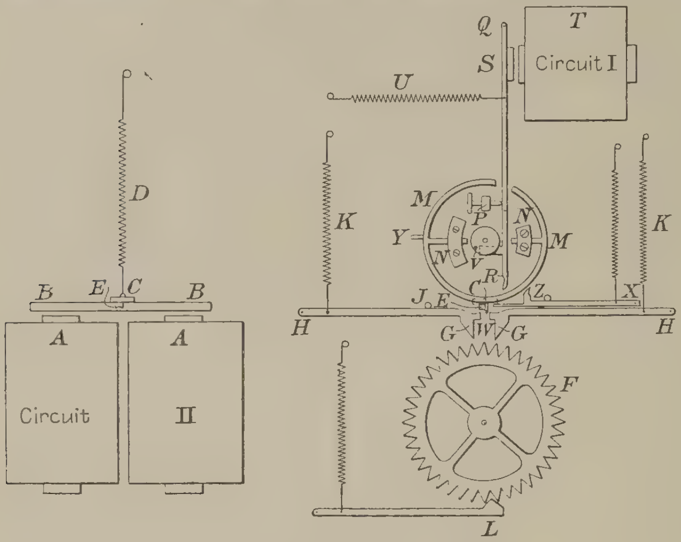


FIG. 6. FIG. 7.

THE SYNCHRONISING MECHANISM.

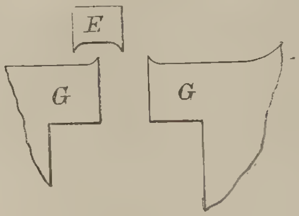
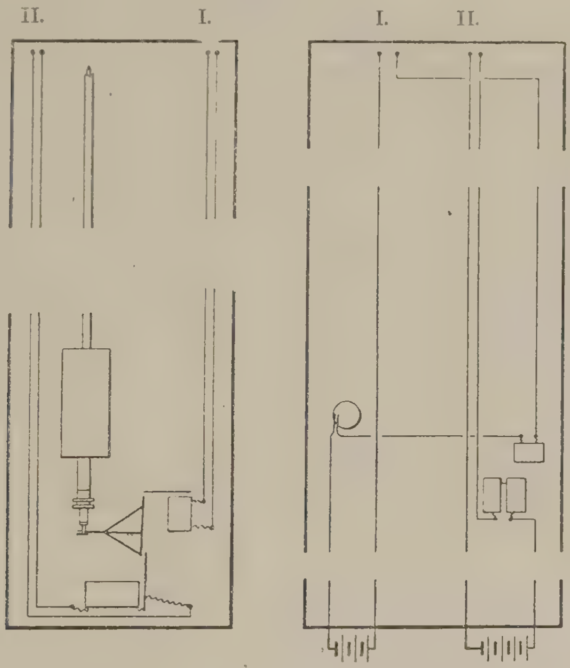


FIG. 8.

by the jumper L. The ends of these pawls are shaped as shown in Fig. 8.

When the lever C is drawn down by the electromagnet A and the tongue E is against its left stop, the latter engages with the left pawl, and the wheel is turned one tooth to the left; similarly, when the tongue is against its right stop the wheel is turned one tooth to the right. At intermediate positions of the tongue one or other of the pawls will generally be engaged; between the ends of the pawls there is but barely sufficient room to let the tongue pass.



Master Pendulum. Slave Clock.

Fig. 9.—The Electric Connections.

Above the star-wheel is the timing wheel M M (Fig. 7). This carries weights, N N, movable along a radius, whereby it is thrown out of balance, and these weights are so adjusted that the period of time the wheel takes to turn from rest through 90 deg. to its position of equilibrium is the period *t* referred to in § 12.

The lever Q R, pivoted at Q, attached to the armature S of the electromagnet T (circuit I.) holds the timing wheel by the tension of the spring U, and through the pressure against the ruby pin V, in a position 90 deg. from its position of equilibrium. The lever Q R is held against a screw stop, P. The pin W on the periphery of the wheel presses the tongue E to its left position. In this position the timing wheel is held during the intervals between successive minute impulses.

15. *Mode of Working.*—At the completion of each minute the counting wheel (Fig. 4) closes circuit I., the impulse lever is released on the pallet of the master pendulum, the lever Q R (Fig. 7) is drawn to the electromagnet, and is held by the detent X Z, and the timing wheel begins to fall. As this wheel turns the impulse lever runs down the impulse pallet. On falling off the pallet the impulse lever closes circuit II., is thrown back, and the lever C (Figs. 6 and 7) is drawn down, releasing the lever Q R. If the two pendulums are in synchrony the timing wheel will have reached its position of equilibrium when this occurs, and the pin Y on its periphery will have pushed the tongue to the right a sufficient distance for the latter to pass between the two pawls G G ; the star-wheel will not be turned, and no change will be made in the rate of the slave pendulum. If, however, the latter is in retard the timing wheel will be released late, the tongue will engage with the left pawl, and the rate of the slave pendulum be increased ; if in advance its rate will be decreased. This automatic comparison and adjustment is made each minute, and the synchrony of the two pendulums maintained as explained in §§ 12 and 13.

The electric connections are shown in Fig. 9.

III. MATHEMATICAL DISCUSSION.

Table of Symbols.

Quantities in C.G.S. units unless otherwise stated.

		Values in model.
$4a$	Latus rectum of parabola (figure of pallet)	$1{\cdot}40$ cm.
E	Total energy of master pendulum	—
ΔE	Energy conveyed by each minute impulse	$1{\cdot}87 \times 10^3$ gm. cm.^{2} sec.$^{-2}$
$\Delta e'$	Change of daily rate, in seconds	—
F	Couple retarding pendulum by wheel friction on pallet	About $2{\cdot}4 \times 10^3$ gm. cm.^{2} sec.$^{-2}$
I	Moment of inertia of master pendulum......	$6{\cdot}47 \times 10^7$ gm. cm.^{2}
i	Moment of inertia of impulse lever..........	$2{\cdot}07 \times 10^2$ gm. cm.^{2}
K	Gravity couple of impulse lever...............	$3{\cdot}69 \times 10g$ gm. cm.^{2} sec.$^{-2}$
L	Length of master pendulum, suspension to pallet	$1{\cdot}19 \times 10^2$ cm.
l	Length of impulse lever, pivots to centre of wheel	$9{\cdot}2$ cm.
N	Change of rate in seconds per day imposed each minute	—

		Values in model.
P	Horizontal component of wheel pressure on pallet	—
s	Angular distance of centre of impulse wheel from beginning of impulse face when $\theta=0$; positive to right	—
T	Semi-period of pendulum	1 second
t	Time interval between the closing of the two electric circuits	—
V	Linear velocity of pallet when $\theta=0$	6·52 cm. sec.$^{-1}$
X	The quantity $\frac{1}{a}\frac{da}{dt}=\frac{1}{\log_{10}e}\cdot\frac{d\log_{10}a}{dt}$: $a=\frac{\pi}{180}$	$1{\cdot}60\times10^{-4}$ sec.$^{-1}$
x_1	The horizontal travel of the pallet while the wheel is rolling down the impulse face	$8{\cdot}3\times10^{-1}$ cm.
a	Semi-amplitude of pendulum in radians	$\pi/180$
θ	Position of pendulum at time under consideration; positive to the right	—

16. *The Figure of the Impulse Pallet.*—With a view to a steady fall of the impulse wheel, the impulse slope is so shaped that the path of the wheel relative to the pallet is a parabola. The wheel then falls with a constant acceleration.* The extent of the impulse is confined to about the middle fifth of the complete amplitude of the pendulum. Within these limits the velocity V of the pallet is nearly constant.

It can be shown that the horizontal component of the force exerted by the wheel on the pallet (*see* Fig. 10) is

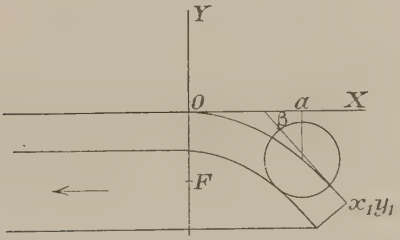


FIG. 10.—THE FIGURE OF THE IMPULSE PALLET.

$$P=\frac{x}{2al}\left(K-\frac{V^2i}{2al}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the energy conveyed to the pendulum with each minute impulse is

$$\Delta E=\frac{x_1^2}{4al}\left(K-\frac{V^2i}{2al}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

---

* In order that the parabolic path may be strictly followed the radius of the wheel must not exceed a certain value dependent on the latus rectum and the constants K and i of the lever.

No special importance is attached to the above figuring of the impulse face. If d is the distance through which the wheel falls and β the angle with the horizontal made by the path of the wheel relative to the pendulum at the end of the impulse, the last equation can be written

$$\Delta E = \frac{Kd}{l} - \frac{V^2 i}{2l^2} \tan^2 \beta,$$

and in this form is applicable to any figuring.

The first term is the energy of fall of the lever and the second the energy of motion with which the lever leaves the pallet.

The second term on the right of equation (2), involving the moment of inertia of the impulse lever, increases with the square of the amplitude. Hence the impulse conveyed to the pendulum is less when the arc is greater. The inertia of the lever, therefore, exercises some degree of control over the arc. As the writer is of opinion that it is advisable that the lever be as light as possible its inertia is made as small as is consistent with a sufficient value of K.

17. *The Energy Required to Maintain the Master Pendulum.*—The energy necessary to maintain the arc at the amplitude decided upon can be found by determining the decrement $d\alpha/dt$ of arc when the pendulum is set to swing freely.

We have

$$\frac{d\alpha}{dt} = \frac{\alpha\, d \log_e \alpha}{dt} = \frac{\alpha}{\log_{10} e} \cdot \frac{d \log_{10} \alpha}{dt}.$$

Log $_{10}$ α is then plotted against time, and a graph obtained which is nearly linear. The tangent at the point corresponding to the amplitude decided upon gives $d \log_{10} \alpha/dt$.

It can be shown that the energy required to be conveyed each minute to maintain the amplitude at α radians is

$$\Delta E = \frac{60}{\log_{10} e} I \frac{\pi^2}{T} \alpha^2 \frac{d \log_{10} \alpha}{dt} . \quad . \quad . \quad . \quad . \quad (3)$$

Combining this with (2) we have

$$\frac{x_1^2}{4al}\left(K - \frac{V^2 i}{2al}\right) = \frac{60}{\log_{10} e} I \frac{\pi^2}{T} \alpha^2 \frac{d \log_{10} \alpha}{dt},$$

from which suitable values of K, i, &c., can be calculated that will give the required arc.

18. *The Couple Retarding the Pendulum Due to the Friction of the Wheel on the Pallet.*—A knowledge of the value of this couple is useful in order that its effects on the rate of the pendulum may be estimated (*see* § 27).

To determine this value the pendulum is swung with the wheel rolling under the weight of the lever on the horizontal portion of the pallet, which is made some centimetres long for this purpose. The decrement of amplitude with time is observed. Under these conditions this decrement is that due to the friction, in addition to that to which the free pendulum is subject. The latter has been found by the method of § 17, and the difference is that due to friction.

If F be the friction couple retarding the pendulum and Δa the change of amplitude due to wheel friction in time T, it can be shown that

$$\Delta a = -\frac{2FT^2}{I\pi^2}.$$

Within the limits of error of experiment, which are somewhat large, this couple is found to be independent of the amplitude.*

19. *The Rating Spring.*—In Fig. 11, A C is the slave pendulum, B E the horizontal connecting rod, D E the suspension wire attached to the rod at E, E F the spiral spring and its connections with the star-wheel. When the pendulum is vertical D, E and F are in a vertical straight line.

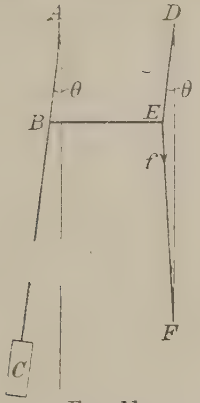


FIG. 11.

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* It may be of interest to refer to the changes in the value of F that were found with the experimental model under different conditions. When the wheel and its jewelled pivots were new and freshly lubricated F had the value $2{\cdot}13 \times 10$ grammes cm.^{2} sec.$^{-2}$. After a year when the pallet was dusty and the pivots not re-lubricated, this figure rose to 2·75; and after cleaning the pallet it fell to 2·39. The figure in the last decimal place is probably of little significance.

Let f be the tension of the spring and M the mass of the slave pendulum bob, and let $DE=AB=l_a$ and $EF=l_b$. Then, if ΔT be the change of period due to a change of tension Δf, we have with sufficient accuracy

$$\Delta T=-\frac{\pi^2\frac{l_a}{l_b}(l_a+l_b)}{2MgT}\Delta f.$$

From this the constants of the two springs to give the necessary change of period can be calculated.

20. The method of synchronising the slave clock with the master pendulum can be explained graphically. In Fig. 12, with the lower curves, ordinates represent rate imposed on the slave pendulum by the rating spring, zero rate being that attained when the two pendulums have the same period. Abscissæ represent time. Rate is imposed at each minute comparison in constant quantity N, positively when the error is negative, negatively when the error is positive. Curve I. represents the hypothetical case in which there is no backlash on reversal in the action of the mechanism altering the rate. In curve II. there is backlash of value $2N$, as shown by the dotted line following behind the full line when a change of direction takes place. In curve III. a negative backlash of value $2N$ imposed by the supplementary spring is shown by the dotted line preceding the full line on change of direction.

The full line represents in the reversed direction the position of the lower end of the principal rating spring.

In each case the initial state ($t=0$) is taken when error$=0$, rate$=5\,N\pm$backlash. The error accumulated (Nt) is represented by the area between the graph and the zero line, changing positively when the graph is above, negatively when below the line.

In curve I., at the end of the first minute, rate being positive, a positive error has accumulated, and consequently the rate falls, and in five minutes a positive error represented by the area a, b, c has been acquired. The rate continues to fall until the positive error has been wiped out, which, from the symmetry of the figure, obviously occurs when it has reached a negative value equal to the positive value from which it started. This hunting of error and rate with constant amplitudes about their zeros, with error a quarter of a period in phase behind rate, will continue indefinitely until a disturbance increases or decreases the amplitude.

In curve II., owing to backlash, the real rate imposed,

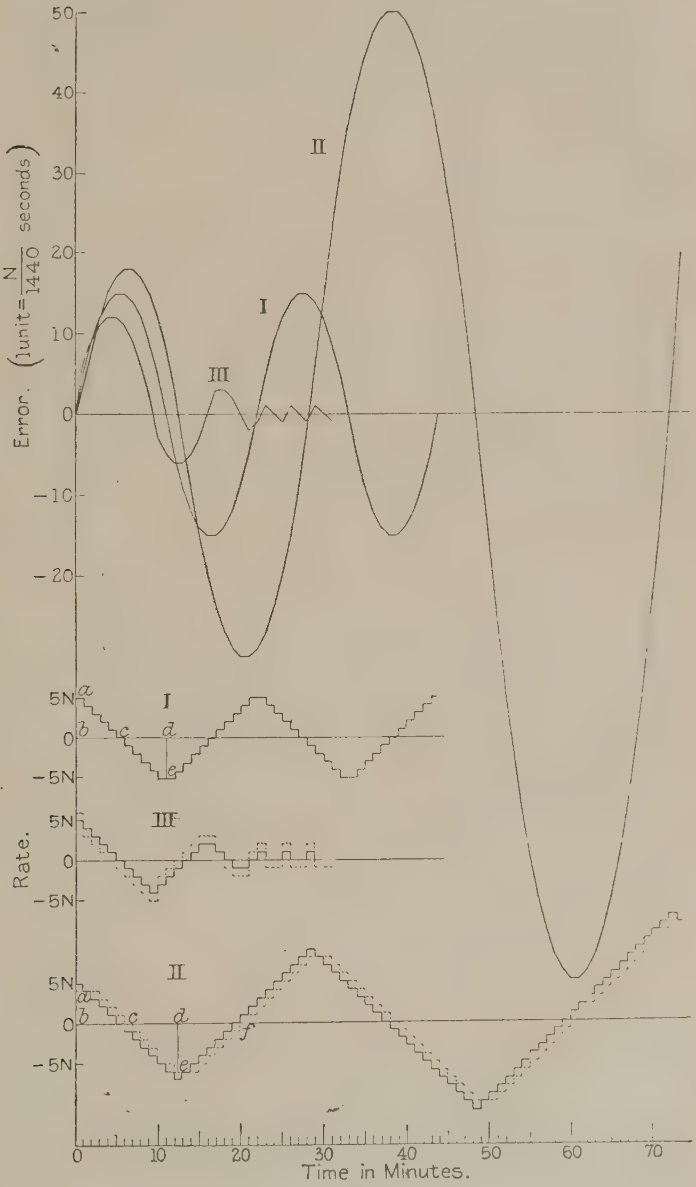
50
40
30
20
10
0
−10
−20
Error. (1unit = $\frac{N}{1440}$ seconds)
I
II
III
5N
0
−5N
Rate.
a
b
c
d
e
f
0
10
20
30
40
50
60
70
Time in Minutes.

FIG. 12.

shown by the dotted line, follows behind the action of the mechanism. Consequently a greater fall in the rate takes place before the area *c*, *d*, *e* equals the area *a*, *b*, *c*—that is,

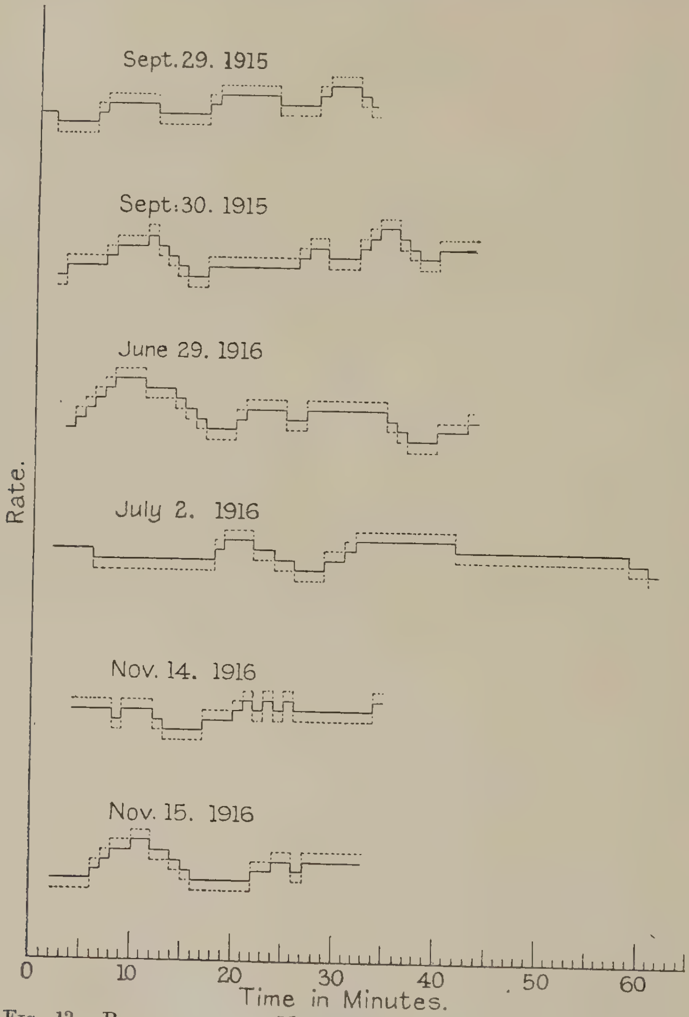

FIG. 13.—RECORDS OF THE MOVEMENTS OF THE RATING SPRINGS OF THE EXPERIMENTAL MODEL.

before the negative rate has wiped out the accumulated positive error. The area *d*, *e*, *f* is again larger for the same reason.

In this way the amplitudes and the periods of the swings grow larger indefinitely.

Curve III. shows the opposite effect of negative backlash due to the supplementary spring, the amplitudes and periods of the changes in rate and error becoming less until they are reduced to small waverings on alternate sides of the zero. These small changes never cease, but the accumulated error at any time is inconsiderable.

The same three cases are plotted in the upper curves, but with ordinates denoting error.

In Fig. 13 are represented some actual records of the movements of the rating springs of the experimental model. Here N is about $\frac{1}{3}$ second per day and the movement of the supplementary spring imposes a rate of $2N$. The zero line changes with changing conditions of the slave clock. There is a small unknown margin of error within which no change of rate takes place. There are therefore periods of varying length during which no change of position of the rating spring is made.*

21. The timing wheel, weighted at one side, is held at 90 deg. from its equilibrium position. It is adjusted so that the time taken in falling through 90 deg is the interval t between the closing of the two electric circuits (*see* §§ 12 and 14). Let I_ω be the moment of inertia of the wheel and $K_\omega \sin\theta$ its restoring couple when displaced θ from the vertical.

The time occupied in falling through θ to the equilibrium position is

$$t=\frac{\pi}{2}\sqrt{\frac{I_\omega}{K_\omega}}\left\{1+(\tfrac{1}{2})^2\sin^2\frac{\theta}{2}+\left(\frac{1\,.\,3}{2\,.\,4}\right)^2\sin^4\frac{\theta}{2}+\&c.\right\} \quad . \quad (4)$$

Putting $\theta=\frac{\pi}{2}$, we have $t=1{\cdot}85\sqrt{\frac{I_\omega}{K_\omega}}$.

---

* With the experimental model the accumulations of error rarely, if ever, amount to ±1/200 second. The slave clock used has a poor train, and its pendulum shows periodic changes of amplitude ; moreover, the synchronising mechanism is somewhat crude, the parts having been designed by estimation and never subsequently altered ; also the value of N and the unit of negative backlash have been made greater than has proved to be necessary. With a better slave clock, having a heavier pendulum, and with improvements in the synchronising mechanism learned by experience to be advisable, the accumulations of error could very probably be made never to exceed ±1/1000 second.

The angular velocity on reaching the equilibrium position is

$$\sqrt{\frac{2K_\omega}{I_\omega}}=\frac{2\cdot 62}{t}.*$$

From (4) we have—

$$\frac{dt}{d\theta}=0\cdot 27\frac{\pi}{2}\sqrt{\frac{I_\omega}{K_\omega}}=0\cdot 27\frac{\pi}{2}\,\frac{t}{1\cdot 85}.$$

Hence, if t is $\frac{1}{4}$ second, a variation of the initial position of the wheel of 1 deg. will cause a variation of the time of fall of only 1/1000 of a second, a point of practical importance.

IV. DISCUSSION OF CERTAIN CAUSES OF VARIATION AND THEIR PROBABLE EFFECTS ON THE RATE.

22. The effect of change of temperature acting directly on the rate of the master pendulum is not considered, as it is assumed that this has been correctly compensated in the pendulum, and that the uniform conditions as regards temperature to which the pendulum is intended to be exposed will render these negligible.

Neither is any consideration given to change of atmospheric density, this, it is assumed, having been eliminated by the enclosing of the pendulum in an airtight case.

The variations requiring consideration seem to be comprised in the following :—Changes in

(*a*) The friction of the wheel on the impulse pallet.

(*b*) The instant of release of the lever on the pallet.

(*c*) The relative position of the pallet and the impulse lever due to tilting sideways of the master pendulum and its mechanism and of the sustaining wall.

(*d*) The conditions of the impulse lever.

(*e*) The effect of the blow of the wheel on the impulse pallet.

These variations may generally affect the rate in two distinct ways, which can be considered separately—viz., (i.) by varying the amplitude and so altering the rate by circular error, and (ii.) by varying the disturbance to the free swing during contact between the wheel and the pallet and so introducing escapement error.*

---

* In the experimental model the radius of the wheel at the point where the pin engages with the tongue (*c*, Fig. 7) is 1·7 cm., and t is about $\frac{1}{4}$ second, hence, the velocity acquired is 18 cm. per second. The dead space within which the tongue can move without engaging either the accelerating or retarding pawl is less than $\frac{1}{3}$ mm., so that the dead interval of time is less than 1/500 second.

23. *Circular Error.*—There is some reason to doubt the strict applicability in practice of the theoretical formula. In the first place a pendulum suspended by a spring does not exactly follow a circular path, and, secondly, with change of amplitude there result changes of non-conservative resistances which probably slightly affect the period. In default of better information we will calculate on the lines of the elementary theory.

Consideration of circular error may be resolved into consideration of changes of total energy of the pendulum.

The semi-period of a pendulum is

$$T=(1+\tfrac{1}{4}\sin^2\frac{a}{2}+,\ \&c.)\pi\sqrt{\frac{l}{g}}.$$

Hence, with sufficient accuracy

$$dT=\frac{T}{8}a\,da. \quad \dots\dots\dots \quad (5)$$

We have seen (§ 17) in dealing with the decrement of amplitude with time due to air-resistance that the expression

$$\frac{1}{a}\frac{da}{dt}=\frac{1}{\log_{10}e}\cdot\frac{d\log_{10}a}{dt}$$

is nearly constant, and a method of determining this was given. Calling this quantity X, we have from (3)

$$\Delta E=60I\frac{\pi^2}{T}Xa^2.$$

Hence $$d(\Delta E)=120I\frac{\pi^2}{T}Xa\,da. \quad \dots\dots\dots \quad (6)$$

Hence, from (5) and (6) a small change $\Delta' E$ in the energy conveyed each minute effects a small change ΔT in the period expressed by

$$\Delta T=\frac{T^2}{960I\pi^2X}\Delta' E,$$

or

$$\frac{\Delta T}{T}=\frac{a^2}{16}\frac{\Delta' E}{\Delta E}. \quad \dots\dots\dots \quad (7)$$

---

* This term is used as a convenient one for change of rate due to change of the disturbance of the free swing, though in the present case there is no escapement.

And the change in the rate due thereto is—

$$\Delta e' = -86{,}400\,\Delta T \quad \ldots\ldots\ldots \quad (8)$$

in seconds per day.

24. *Escapement Error.*—The loss or gain of time ΔT in a single vibration due to a disturbance at a point θ, thereby changing the energy of the pendulum by $\Delta' E$ can be shown to be expressed by

$$\Delta T = \pm \frac{T^3}{I\pi^3} \frac{\theta}{\alpha^2\sqrt{\alpha^2-\theta^2}} \Delta' E,\text{*} \quad \ldots\ldots \quad (9)$$

the positive sign applying to the case where the disturbance occurs when the pendulum is rising, the negative when falling, θ being positive.

If this change of period occurs with each minute impulse, the change of rate due thereto is

$$\Delta e' = -1{,}440\,\Delta T \quad \ldots\ldots \quad (10)$$

in seconds per day.

25. The application of the above expression for escapement error to friction of the impulse wheel is of importance and will be required in the discussion to follow.

From the definition of the frictional retarding couple (§ 18)

$$dE = \mp F d\theta,$$

the upper sign applying when the pendulum is rising and θ is positive, whence from (9)

$$dT = -\frac{T^3 F}{\pi^3 I} \frac{\theta}{\alpha^2\sqrt{\alpha^2-\theta^2}} d\theta.$$

In integrating this expression it is to be noted that both α and θ are variable. With such values as F is likely to have the error involved in treating α as constant is negligible. The change of period of an individual vibration due to friction of the wheel between θ_0 and θ_1 is then

$$\Delta T = \frac{T^3 F}{\pi^3 I \alpha^2} \left\{ \sqrt{\alpha^2-\theta_1{}^2} - \sqrt{\alpha^2-\theta_0{}^2} \right\} \quad \ldots \quad (11)$$

26. The application of the same expression (9) to the disturbance of free-swing brought about by the impulse will also be required. This will depend on the position of the pendulum at which the impulse is adjusted to take place.

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* *See* G. B. Airy, "On the Disturbances of Pendulums, &c.," Camb. Phil. Soc. Trans., Vol. III., 1830, p. 105; J. M. Bloxam, Memoirs A.S., Vol. XXII., 1853.

In Fig. 10 let the position of the wheel as drawn represent that which it occupies on the pallet when the pendulum is at its vertical position. The pallet is moving to the left, and the wheel is rolling down the impulse face from $x=0$ to $x=x_1$. Let s be the angular distance $0a$.

It can be shown from (1) and (9), the positive direction of θ being taken towards the right, that the change of the individual period due to the element of impulse through $d\theta$ is

$$dT=\frac{2T^3L^2}{\pi^3 I}\frac{\Delta E}{x_1{}^2}\frac{\theta(s-\theta)d\theta}{\alpha^2\sqrt{\alpha^2-\theta^2}},$$

and the whole change of period due to the impulse, treating α as constant, is

$$\Delta T=\frac{T^3L^2}{\pi^3 I}\frac{\Delta E}{x_1{}^2}\cdot\frac{1}{\alpha^2}\left\{\alpha^2\sin^{-1}\frac{s}{\alpha}-\alpha^2\sin^{-1}\frac{s-\frac{x_1}{L}}{\alpha}\right.$$
$$\left.+s\sqrt{\alpha^2-s^2}-\left(s+\frac{x_1}{L}\right)\sqrt{\alpha^2-\left(s-\frac{x_1}{L}\right)^2}\right\} \quad . \quad . \quad (12)$$

Owing to the greater forces acting in this case the discrepancy involved in assuming the constancy of α is appreciable. In the application in § 20 the value of ΔT obtained from the use of this expression was found to differ by about 2 per cent. from the rigid value.

27. We are now in a position to consider the effect on the rate of the clock of the changes set out in § 22. In order to do this the data supplied by the experimental model will be used (*see* Table of Symbols).

(*a*) *A Change in the Value of the Friction of the Wheel on the Impulse Pallet.*—(i.) *Circular Error.*—Assuming that friction is proportional to the varying pressure, and neglecting the inertia term in (1), we find (*see* §§ 18 and 25) the change of energy of the pendulum due to friction during one impulse is

$$\Delta_2 E=F(\theta_3-\theta_1),$$

where θ_1 is the position when the lever falls on, and θ_3 when it falls off, the pallet.

Hence, the change due to change of friction is

$$\Delta' E=\Delta F(\theta_3-\theta_1).$$

And from (7) and (8) the change of daily rate is

$$\Delta e'=-86{,}400\frac{T\alpha^2}{16\Delta E}\Delta F(\theta_3-\theta_1).$$

Putting $\theta_1=0\cdot3\alpha$, and $\theta_3=-0\cdot25\alpha$, as in the adjustment made in the model; and putting $\Delta F=6\cdot2\times10^2$, probably an extreme value (*see* Footnote to § 18), we find

$$\Delta e'=+0\cdot0052 \text{ second per day.}$$

(ii.) *Escapement Error.*—Using the same value for the change of friction, we have from (10) and (11)

$$\Delta e'=-0\cdot00036 \text{ second per day.}$$

28. (*b*) *A Change in the Instant of Release of the Impulse Lever.*—Such changes inevitably occur by the nature of the mode of synchronising, but they are periodic, and their effects are not cumulative. Probably an extreme value of such a change, taking one whole day with another, would be ±1/100 second. The normal value $\theta_1=0\cdot3\alpha$ becomes $\theta_1=0\cdot3\alpha\pm0\cdot03\alpha$.

This change may conceivably affect the rate by (α) varying the length of pallet along which the wheel rolls, whence by (i.) circular error

$$\Delta e'=-0\cdot0011 \text{ second per day,}$$

and by (ii.) escapement error

$$\Delta e'=+0\cdot0009 \text{ second per day.}$$

(β) This change may also affect the rate by altering the position in the arc at which energy is drafted by starting the wheel or by the blow on the pallet. The effect of the former is negligible, and that of the latter is dealt with in § 31.

29. (*c*) *A Shift of the Point of Suspension Sideways Relatively to the Impulse Lever.*—Such a shift or tilt might arise from a want of stability of the wall to which the master pendulum was attached, or from a faulty attachment of the pendulum or the impulse mechanism to such wall.

(α) The immediate effect will be to alter the value of s (*see* § 26). Differentiating the expression (12) with respect to s, we get by escapement error

$$d(\Delta T)=\frac{2T^3L^2\Delta E}{\pi^3 I x_1{}^2\alpha^2}\left\{\sqrt{\alpha^2-s^2}-\frac{\alpha^2-s^2+\frac{x_1}{L}s}{\sqrt{\alpha^2-\left(s-\frac{x_1}{L}\right)^2}}\right\}ds\quad. \qquad (13)$$

Putting, as in the model, $s=0\cdot15\alpha$, and giving ds the value 1′ of arc, we find

$$\Delta e'=0\cdot076 \text{ second per day.}$$

This is a large variation; moreover, since $d^2\Delta T/ds^2$ is very small with all possible values of s, this figure cannot be materially reduced by altering the position in the arc at which the impulse is given.

This investigation points to the necessity of firm attachment of the pendulum and its mechanism to their support. The same effect would obtain with the Graham dead-beat and the generality of escapements, but with those where the impulse is given symmetrically in two directions the effect in one direction is compensated by the counter effect in the other when the tilt is small.

This effect is not a consequence of the particular way in which the impulse is given. The most ideal impulse where all the energy is given at the position $\theta=0$ would yield an almost identical figure.

(β) Another effect of such a tilt will be one similar to that dealt with in § 28 (*b*), but the changes of rate are smaller than there resulting.

30. (*d*) *A Change in the Conditions of the Impulse Lever.*—(α) One of the conditions here to be considered is that of friction at the pivots. Such friction will affect the rate by altering the value of the impulse.

In order to gain some knowledge of the pivot friction a simple experiment was made. The pivots and their holes had not been cleaned for about two years. The static friction was found to amount to about $K/400$.

(i.) *Circular Error.*—We have from (2), neglecting the inertia term,

$$\frac{\Delta' E}{\Delta E}=\frac{\Delta K}{K},$$

and from (7) and (8)

$$\Delta e'=-86{,}400\frac{T\alpha^2}{16}\,\frac{\Delta K}{K}.$$

Assuming an increase of friction amounting to 100 per cent. of its value, we get

$$\Delta e'=+0{\cdot}0041 \text{ second per day.}$$

(ii.) *Escapement Error.*—Applying the formula (12), putting $s=0{\cdot}15\alpha$, we get, with the same increase of friction,

$$\Delta e'=+0{\cdot}0013 \text{ second per day.}$$

It may be pointed out that by altering the position of the pendulum at which the impulse is given the disturbance to the pendulum by the impulse may be reduced or made to vanish, and changes of this disturbance with change of impulse to vanish also. Thus, if $s=0{\cdot}27\alpha$, ΔT of expression (12) becomes zero. As, however, the absolute value of this disturbance is of no account if it remains constant, as the probable variations of the impulse are small, and as a higher value of s involves the pushing of the point at which the lever falls further from the zero position, it has been judged advisable to give s the value $0{\cdot}15a$.

(β) Another condition of the impulse lever is that of temperature. It is assumed (*see* Introduction) that the changes of temperature to which the master pendulum are exposed are small. A small change of temperature, however, may have a serious effect if it alters the position in the swing at which the impulse is made. Suppose, for instance, that the pivots of the lever remain in constant relation to the pallet while the steel lever expands, then s will change by the quantity

$$\Delta s = -\frac{l}{L} \times 0{\cdot}000011 \times 5$$

for an increase of 5°C., putting 0·000011 as the coefficient of expansion of steel.

From (13) and (10) the change of rate will be

$$\Delta e' = -0{\cdot}0011 \text{ second per day.}$$

31. (*e*). *A Change in the Effect of the Fall of the Wheel on the Impulse Pallet.*—In order that the blow of the wheel on the pallet shall be small it is essential that the wheel shall be held but a fraction of a millimetre above the pallet. This involves the rigid attachment of the impulse mechanism to the suspension. It is proposed to suspend the pendulum in a cast-iron box with a plate-glass window, and to attach the impulse mechanism to the cast iron. The relative change of vertical position of the wheel and pallet due to expansion through 5°C. would be about 0·0055 cm.

Careful consideration has been given to the question whether the impulse should not preferably be given at a point higher up the pendulum with a view to diminishing any relative change in position of wheel and pallet. It appears that any advantages of so doing are outweighed by the difficulties and disadavantages.

With a view to measure the effect of the blow a series of experiments were made, in each of which during a measured time (20 minutes) the wheel was dropped a large number of times (337 to 400) through a distance of 0·02 cm., and, after a short measured run along the horizontal pallet, was raised. The decrement of amplitude was read. The experiments were rough, but within the probable limits of error the decrement of amplitude was fairly closely accounted for by the air resistance and the frictional resistance of the run. The resistance to the pendulum due to the blow of the wheel seemed very small.

Table of Resulting Changes of Rate.

Due to—	By circular error.	By escapement error.
	Second	per day.
(*a*) Change of friction of impulse wheel (25 p.c.)...	±0·0052	∓0·00036
(*b*) Change of instant of release (0·01 second) of impulse lever	±0·0011	∓0·0009
(*c*) Tilt of master pendulum attachments (1′ of arc)	...	±0·076
(*d*) (α) Change of friction at impulse lever pivots (100 p.c.)..	±0·0041	±0·0013
(β) Expansion or contraction of impulse lever through 5°C.	...	∓0·0011

The change in each case is taken as acting through one whole day.

It may appear that in the course of this discussion of probable causes of variation of rate, the mathematical mill has been applied to grind out results of greater refinement than is consistent with the trustworthiness of the data that can be used. Considering, however, the simplicity of the conditions to which this pendulum is exposed, this mathematical application seems to the writer to be justified; he believes that these results give useful evidence of the order of magnitude of the variation of rate to be expected from the various causes considered. It is possible that other causes of variation that have not occurred to him may be found to be of equal or greater importance.

32. *The Best Value for the Amplitude of the Master Pendulum.*—The degree of freedom from frictional disturbance and the confining of the impulse and all mechanical contact to a small angle on each side of the vertical position lead one to infer that escapement errors will be of less importance as compared with circular errors in the case of this clock than usually obtains, and this consideration seems to justify the adoption of the unusually small semi-amplitude of 1 deg.

The following table has been calculated for a semi-amplitude of 2 deg., certain reasonable and convenient necessary modifications of other conditions having been made. A comparison of this table with that of § 31 will illustrate the argument used above.

It is anticipated that the error due to the change (*c*) can be reduced to small value :—

Table of Changes of Rate with Semi-amplitude doubled (2 *deg.*).

Due to—	By circular error.	By escapement error.
	Second	per day.
(*a*) Change of friction of impulse wheel (25 p.c.)...	±0·006	∓0·0002
(*b*) Change of instant of release of impulse lever (0·01 second)	±0·002	∓0·0003
(*c*) Tilt of master pendulum attachments (1′ of arc)	...	±0·038
(*d*) (α) Change of friction at impulse lever pivots (100 p.c.)...	±0·016	±0·0006
(β) Expansion or contraction of impulse lever through 5°C.	...	∓0·0006

V. Conclusion.

It is unusual to put forward a new piece of physical apparatus without any attempt to report as to its efficiency in practice. But in this case the conditions are unusual. No comparison of a clock of the precision to which this aims is of any use except with the daily records of a first class observatory. The Paris rhythmic time signals have not been available to private observers during the war. Moreover, the purpose of the experimental model is merely to show the practicability of the method of synchronising the two pendulums and of the mode of giving the impulse to the master pendulum ; to show that the clock works without risk of failure of the electric contacts or other details. This the writer claims has been achieved. He suggests that, the practicability of the method being shown, the conditions for good timekeeping are assured.

The model exhibited has involved considerable expenditure, and the writer feels justified in bringing the subject forward in its present stage in the hope that he may be sufficiently encouraged to expend greater sums on an instrument capable of undergoing the severest tests.

Warm acknowledgment is due by the writer to Mr. Joseph Kirner for the skilful way he has carried out the ideas embodied in this piece of apparatus, for the many valuable practical suggestions he had made, and for the great interest he has shown.

The writer is much indebted to Prof. Alfred W. Porter for kindly reading the manuscript, and to Prof. C. V. Boys, who has read the Paper and advised as to its publication.

ABSTRACT.

The principal feature of the author's arrangement is the employment of a "slave" clock to do the great part of the work, leaving the master pendulum no function beyond that of controlling the rate of the other. The master pendulum swings freely except for a short period (about one-fifth of a second) every minute, during which it receives an impulse from a small falling wheel electromagnetically released by the slave clock. At the end of its fall the impulse mechanism closes a second circuit and is restored to its initial position. These two electric circuits also energise parts of the mechanism in the slave clock by which the latter is kept in time with the master pendulum. The lagging of correction behind error, with the resulting periodic fluctuation in the rate, is reduced almost to the vanishing point by the introduction of a "negative backlash" in the control mechanism.

The Paper contains a mathematical discussion of the best working conditions and of the possible magnitude of errors which might arise from various causes.

DISCUSSION.

Prof. Boys contributed the following remarks : The subject with which Mr. Bartrum has dealt with such ingenuity and success is one which has the greatest fascination for me, and I am sure that the members of the Society will join with me in congratulating him. I feel that we should welcome this contribution the more because the author's daily work is quite in another walk of life. He has referred to the late Sir David Gill's clock with two pendulums, which I saw when I was at Capetown, and which Sir David explained to me very fully. While admiring the ingenuity of this, I felt that it was not a convenient solution of the general problem. In particular I distrusted the use of a pendulum hooked up and liberated every two seconds, so that the slave clock, in fact, was started and stopped 30 times a minute by the master pendulum. Mr. Bartrum has undoubtedly worked on safer and more practicable lines in employing for the slave clock a complete going clock which is only rated instead of being kept alive by the master pendulum, and the less frequent interactions, which are sufficient, are certainly much more desirable. Of the numerous elements which go to make up the complete device, I think that which I most admire is the device for negative back lash, and the short account of this and the purpose that it serves would I think, be read with advantage by anyone who has to do with governors of any kind, and of the difficulties due to hunting which are met with in such varied directions. The author has realised with very great perfection the ideal conditions of a free pendulum which may be placed 60 ft. below the ground in a place of uniform temperature and within a casing, so as to be protected from variations of atmospheric density, whether due to change of temperature, if any, or of barometric pressure. Such a pendulum, free from mechanical friction variations, and with a gravity impulse related exactly as wished to the centre position, should give the most perfect time keeping that can be realised. At the end of the Paper the author refers to the question of the most suitable arc of vibration. It is possible that this can only be determined experimentally, as it has been with escapement clocks, for there are here as elsewhere the counteracting advantages of improving isochronism with reduction of arc and improving stability of amplitude with increase of arc. However, such tests could only be made usefully under the most perfect conditions for the master pendulum, which up to the present the author has not been able to realise. He has, however, shown that his device will work, and that one pendulum will govern that of the other clock for some two years without trouble, and I consider that he has proved his proposition. This is not a mere paper invention full of ingenuity which, to use a common but erroneous expression, is all right in theory but in practice won't work. In practice it does work, and it does not appear to me to be as a corollary all wrong in theory. When any inventor breaks up new ground and brings to light new material of value he is apt to be shadowed by a host of minor inventors who very possibly may make some improvements in detail. In the present case it is certain that the comparatively small number of people who are interested in the production of the perfect clock will see other ways of doing what Mr. Bartrum has done. In fact, each one, if he thinks about it, will probably devise two or three different ways, and among all these, bits here and bits there, may very well be combined to produce a more convenient design. But there is no particular merit in doing this ; the merit is in the device which others seek to modify rather than in the modifications. It is rare that there is so much merit in a parody as in an original composition. I wish to lay stress on this, because after seeing Mr. Bartrum's clock I found it impossible to avoid thinking of other devices. These were mainly due to features in Mr. Bartrum's design which I did not quite like. For instance, that against which my mechanical sense most rebelled was the labour imposed upon the slave pendulum of dragging round the timing wheel. I also disliked the idea of the springs and connections to the pendulum of the regulating gear. I feel that these impose such a burden on the slave pendulum as possibly to make regulation far more frequently needed than it would be if this pendulum were as untramelled as that of an ordinary regulator. In other words, that the pendulum is truly described as a slave rather than as a colleague. I wanted, therefore, to see if the regulator clock could not be left just as free and undisturbed as any regulator, for in that case the problem of governing it should be simplified. For instance, there are 86,400 seconds in a day, and an uncertainty of one second a day should not be possible in any such type of clock. A correction of rate of 1/80,000 in round figures is all that can ever be needed to keep the colleague clock in order. Such a correction is effected by attaching to the pendulum rod half way down to the centre of oscillation a weight of 1/10,000 of that of the bob. In any one minute the cumulated effect of such a change of rate is about 1/1,400 second. If, therefore, a bicycle ball of convenient weight can be tipped into a hole in a shelf half way down or lifted out again we have sufficient correction for any possible variation of rate, and with this alone, which is Mr. Bartrum's negative back lash, and no other correction at all, the necessary governing can be done, and never a 1,000th of a second divergence arise between the pendulums.

In the master pendulum case, which it ought not to be necessary to visit more often perhaps than once in 20 years, there should be no pivots and no lubrication. The clock maker dotes on pivots, and here I think Mr. Bartrum has come unduly under his influence. I think the impulse could better be given by a weight resting upon an arm carried by and one with the pendulum on one side of the suspension spring, with an effective fall of precisely limited extent, and operating once a minute. An electromagnetic device for holding this weight out of the way or for placing it in a defined position, so that it may be picked up by the rising arm and arrested in a lower position from the falling arm is easily designed, and if here a pivot should be found convenient it need not involve lubrication.

The timing wheel and its operation is one of Mr. Bartrum's devices which I admire, but, together with the counting wheel, it can be abolished altogether if the two pendulums are made synchronous in phase and kept so. In order to make such a method possible I would suggest that the escapement wheel of the colleague clock should make a relay contact when the clock ticks which should cause a spring supported plate below each pendulum to rise into operative position and there be held for about one second. Each of these carrying a rod of Acheson graphite would then make contact with fine wires carried at the ends of the pendulums just as they pass their middle positions. These two contacts would one of them send a current to a racing relay if the other had not sent one first. Whichever one, therefore, won the race, and got in first, would cause the relay to send on the appropriate one of two currents, one to put the ball on the regulating shelf of the colleague pendulum and the other to take it off. The colleague clock would also make a contact in the pendulum plate in plenty of time to cause the impulse weight to be put in its place ready to be picked up by the master pendulum, and another one to elevate it out of reach when it had done its work. It seems quite unnecessary to describe the very simple electromagnetic devices needed.

I should put a second pendulum plate near the end of the swing of the master pendulum, which the operator could lift for a couple of seconds once a day or once a month as found desirable, so that four successive contacts could be recorded on a chronograph from which the amplitude could at once be determined. I should also bring up two pipes from the bottom and the top of the master pendulum case to the observatory, where a pressure gauge would show the pressure, and so that if it were desired to regulate to a very small extent the rate of the master pendulum this could be done by small variations of pressure, or better by passing to the bottom of the case a known quantity of dry hydrogen gas, or to the top of the case dry heavy gas such as CO_2, so as to alter the density of the surrounding medium to a known but small extent. In normal times I could have made in a few days all that would be necessary to put these suggested modifications of Mr. Bartrum's invention into practice, and I should certainly have done so, as the problem is to me one of very great interest, but it is not possible now to find time for such excursions, and so I have put these forward as suggestions only. I had hoped to have been able to make some experiments in contacts between Acheson graphite and tungsten or tantalum wires, using a number of specimens of these wires, which the British Thomson-Houston Company have kindly given to me for this purpose, but at present this has not been possible. However, in these days contact difficulties are not what they were, if, indeed, they exist at all. The automatic telephone exchange and many other things would wholly fail if contacts were the bogey that they used to be considered.

Mr. DUDDELL said that at the Paris Observatory there was a standard seconds pendulum in a chamber 60 ft. below the surface of the ground. The temperature was constant, and there were no pressure variations, as the pendulum was in a vacuum. It was driven electrically, and maintained a perfectly constant rate, requiring no attention whatever. It gave a signal in a telephone every second.

The AUTHOR, in a communicated reply, said that he agreed that only experience could show the best amplitude to adopt. The President had pointed out that if the time-keeping conditions of the auxiliary clock were so good that its possible variations were small, the control of the master pendulum might be limited to the imposition of two rates, respectively above and below the true rate. This, the author thinks, contains a valuable suggestion. He points out that the master having such limited influence over its colleague, these rates would require to be carefully selected to avoid the danger of the colleague becoming beyond control. The further suggestion that the two pendulums should be maintained without difference of phase involves an electric contact being made by the master, an arrangement the author had been anxious to avoid from fear of its probable variability. If modern materials would enable such a contact to be made safely important simplifications could be effected.

VIII. *The Effect of the Water-vapour in the Atmosphere on the Propagation of Electromagnetic Waves.* *By* FREDERICK SCHWERS, *D.Sc.*

RECEIVED OCTOBER 30, 1916.

I PROPOSE in the following pages to discuss a point in connection with the propagation of electromagnetic waves in space. It seems well founded that the propagation of these waves is strongly influenced by the conditions of ionisation of the air; yet a quantitative theory has not been established since the actual facts—namely the change of intensity at the receiver from day to night, during solar eclipses, &c.—are not known with sufficient accuracy.* For the present, however, let us disregard the effect of ionisation and consider only the refraction of these waves through the atmosphere.

Suppose our sending and receiving stations are on the opposite shores of an ocean, so that they are separated by salt water only; in this case, the energy sent from an antenna, as shown by Sommerfeld,† is propagated for the larger part in the form of space waves, and only in smaller part in the form of surface waves.

Kiebitz ‡ has made an attempt to calculate the influence of the atmosphere in refracting the space waves towards the surface of the earth. Considering a layer of air 1 km. thick, he found that the refraction produced in this medium of feeble dielectric constant is quite insufficient to bring all the spacial waves down to the surface of the earth; it would thus appear that refraction through the medium air exerts but a small favourable influence on the intensity at the receiving antenna. Considering further the influence of moisture, Kiebitz found that it only increases the dielectric constant by 10 per cent., *i.e.* only to a small extent. In this last respect the calculation of Kiebitz seems to me quite erroneous. Without considering the experimental figures for the dielectric constant of water-vapour, he simply assumed that the expression of Clausius-Mosotti $\frac{k-1}{k+2} \cdot \frac{1}{d}$ remains constant when passing from the liquid to the gaseous state. If we put for liquid water at $T=293°$ $k=81$, we find for water-vapour at 293° and 760 mm. the value $k=1{\cdot}0022$, and with this figure Kiebitz made his further calculations.

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* *See* E. G. Eccles, "Jahrb. f. Drahtl. Telegr.," Vol. VII., p. 191 (1913).

† "Annalen der Physik" (IV.), Vol. XXVIII., p. 665 (1909).

‡ "Jahrb. f. Drahtl. Telegr.," Vol. VII., p. 154 (1913).

As a matter of fact, it can easily be shown by a few examples that the Clausius-Mosotti formula does not remain constant when passing to another state of aggregation of a substance. For the gaseous state particularly, we find very high values of $A\left(=\frac{k-1}{k+2}\frac{1}{d}\right)$, especially when the dielectric constant in the liquid state is already very high. The following table gives a few figures in this respect* :—

TABLE I.

—	T.	A liquid	T.	A gas
	Deg.		Deg.	
Ethyl alcohol	287·7	1·12	383·0	1·47 (Baedeker)†
Acetone	185·4	1·01	...	...
	273·0	1·06	290·3	3·54 (Pohrt) ‡
	291·8	1·12	382·3	2·77 ,,
Ammonia	288·0	1·35	291·4	3·40 (Baedeker)
	...	...	292·0	3·29 ,,
	...	...	332·4	2·91 ,,
	...	...	335·1	2·88 ,,
	...	...	356·8	2·75 ,,
	...	...	368·3	2·67 ,,
	...	...	381·4	2·65 ,,
Water	273·0	1·03	413·0	4·54 (Baedeker)
	283·0	0·98	415·2	4·57 ,,
	...	...	416·2	4·39 ,,
	...	...	418·8	4·16 ,,
	...	...	421·6	3·90 ,,

The A-values for water and ammonia in the gaseous state are plotted on diagrams I. and II. Ammonia is interesting as compared with water, since the values extend over a much wider range of temperature. Baedeker states that in both cases experiments could not be made at lower temperatures because of the conductivity of the gas ; in fact, we notice that for water as well as for ammonia the two first values are somewhat off the general curve.

If for water-vapour we extrapolate down to $T=293°$, by assuming the straight line given by the diagram, we arrive at $A=16·45$, which gives $k=1·037$ for 760 mm., thus much more than the previously calculated figure 1·0022. It must be admitted that such an extrapolation is hazardous ; there is every reason to believe that the temperature function of A runs, not according to a straight line, but according to a curve, the convexity of which is turned towards the abscissa, as in the case of ammonia vapour. For the latter $\frac{dA}{dT}$ increases from 0·005

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* We consider the values of k for large λ, thus practically for $\lambda\ \infty$.

† "Zeitschr. Phys. Chemie.," Vol. XXXVI., p. 308 (1901).

‡ "Ann. der Physik," Vol. XLII., p. 569 (1913).

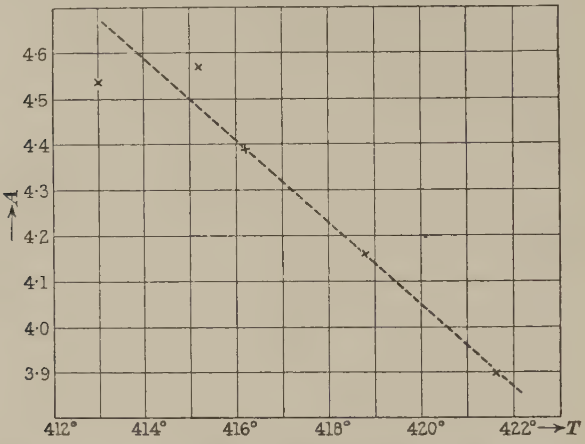

Diagram I.—Variation of the Clausius-Mosotti Expression (A) for Water-vapour as a Function of the Temperature.

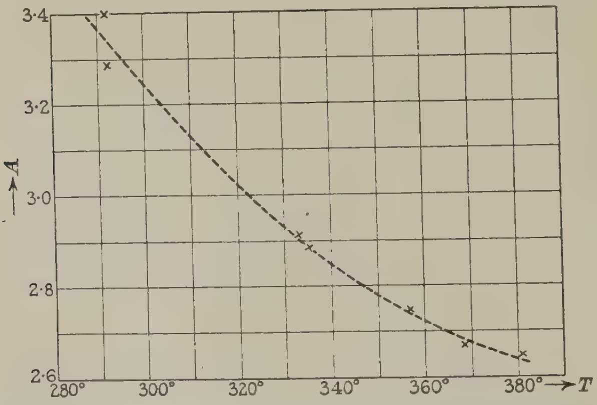

Diagram II.—Variation of the Clausius-Mosotti Expression (A) for Ammonia-vapour as a Function of the Temperature.

to 0·012 for a temperature decrease of 90° only. Yet, since we have no means of ascertaining from the present data the accurate shape of the curve in the case of water-vapour, we shall admit our straight-line extrapolation, without forgetting that the values of the dielectric constant thus obtained are almost certainly too low.

Let us now calculate the refraction undergone by electric waves in the atmosphere. Consider above the earth a layer of air having the height h. In order that the waves may follow exactly the curvature of the earth, the front ab, which is normal to its surface, must be displaced so that cd be also perpendicular at the point d (*see* Fig. 1).

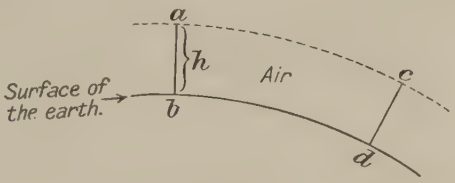

FIG. 1.

If we put r=the radius of the earth (mean= 6366·2 km.), this condition will be satisfied if the relative velocities $\frac{bd}{ac}$ are as $\frac{r}{r+h}$ or, as Kiebitz puts it, as $\frac{40,000}{40,000+2\pi}$.

We must now assume certain definite and regular conditions of the atmosphere—as far as possible the mean conditions that prevail. We suppose, as stated before, that the sending and receiving stations are separated by an ocean and that at sea-level (h_0), the pressure is 760 mm. At h_0 we can admit successively different temperatures according to the latitude. For the change of temperature with increasing h it is somewhat difficult to take an average value. I propose to consider a rather small positive temperature gradient for the first kilometer, say a decrease of 0·2 deg. for every 100 metres, on account of the fact that there is sometimes a negative gradient; in the second and third kilometre I propose to consider larger values of the gradient, say 0·5 deg. and 0·55 deg. respectively.

The relative humidity of air above the oceans will be put at 80 per cent.* For example, at 29·5° 1 kg. of moist air contains 19·53 gr. of water-vapour, when completely saturated; on our hypothesis it will contain only 15·62 gr. (or 0·01562 kg.). From the known relation $0{\cdot}623e/760-0{\cdot}377e=0{\cdot}01562$ we deduce the value of the vapour pressure e as 18·82 mm.

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* Hann, *loc. cit.*, p. 234.

For other values of h, the water vapour pressure diminishes above the oceans according to Suering's empirical formula * : $e_h=e_0 10.^{\frac{h}{6}\left(1+\frac{h}{20}\right)}$. In the case just considered (temp. $298^\circ_{abs.}$ at $h=0$) this would give for $h=1$ km., the value $e_h=$ 12·52 mm., corresponding to 11·58 gr. water-vapour in 1kg. of air. At this height the temperature would be 296°·0 (temperature gradient 0·2°) and the total pressure P, calculated from the well-known formula : $\log 760/P=h/18{,}400(1+at)\ (1+0{\cdot}377\, e/P)$ would be 677·9 mm. Hg. Under such conditions of temperature, moisture and pressure the density of air is 0·001173 at 298° and 0·001056 at 296°. For other values of h we get the figures indicated in Table II.

TABLE II.

h	$T°k$	P	e	Specific moisture.	Density (moist air).	A (H_2O vapour).	k (moist air).
	Deg.						
0	298·0	760·0	18·82	15·62	0·001173	16·00	1·001407
0·5	297·0	719·9	15·47	13·53	0·001114	16·09	1·001230
1·0	296·0	677·9	12·52	11·58	0·001056	16·18	1·001070
2·0	292·5	603·5	8·11	8·43	0·000954	16·50	1·000830
3·0	287·0	536·3	5·01	5·84	0·000863	17·00	1·000650

Let us now consider the dielectric constant. For dry air we have $k=1{\cdot}000590$ † at 298° and 760 mm. For other temperatures and pressures within the range we have to consider, it has been shown that A remains absolutely constant. Thus the value of A for air (dry) will be put 0·152 in all cases. By means of the formula for binary mixtures,‡ we can calculate the value of the dielectric constant of air with 80 per cent. moisture at 298° and 760 mm. We get

$$\frac{k-1}{k+2}\cdot\frac{1}{0{\cdot}001173}=0{\cdot}152\times0{\cdot}98438+16{\cdot}0\times0{\cdot}01562.$$

From this we find $k=1{\cdot}001407$. If we calculate in the same way for other values of h, we get the figures of the last column of Table II. As just said, we may use a constant value for air (0·152) ; but for water-vapour we must take the values from the extrapolated curve, which are to be found in column 7.

* "Wissensch Luftschiffahrten," Vol. III., p. 157 (1900).

† Boltzmann, "Pogg. Ann.," Vol. CLV., p. 403 (1873), and others.

‡ *See* Graetz, "Handb. der Elektr. u. des Magn.," Vol. I. (2), p. 175 (formula 81).

We have now to calculate, taking $k=1\cdot001407$ for the atmosphere at sea-level, what must be the values for different h's in order that the whole of the electromagnetic waves may be refracted along a curve corresponding to that of the earth's surface. Admitting that k is the square of the electric refractive index, we have that the velocity of propagation expressed in km. is equal to the velocity in vacuo divided by $\sqrt{k}$, thus $\frac{3\cdot10^{-5}}{\sqrt{(1\cdot001407)}}$ or $\frac{3\cdot10^{-5}}{1\cdot0007035}$ for $h=0$.

For any other value of h we have

$$\frac{6366\cdot2}{6366\cdot2+h}=\frac{\dfrac{3\cdot10^{-5}}{1\cdot0007035}}{\dfrac{3\cdot10^{-5}}{\sqrt{k_h}}}=\frac{\sqrt{k_h}}{1\cdot0007035}.$$

If we calculate $\sqrt{k_h}$ for the values of h considered and take the square, we get the figures under k' in Table III.

TABLE III.

h	k	k'	$k'-k$
0	1·001407	[1·001407]	...
0·5	1·001230	1·001250	+0·000020
1·0	1·001070	1·001092	+0·000022
2·0	1·000830	1·000778	−0·000052
3·0	1·000650	1·000464	−0·000186

We see that the figures under k, representing the actual dielectric constant, are first lower then higher than the figures under k'. If we put the difference between k and k' as ordinates and h as abscissa, the maximum of the curve will correspond to the height below which all waves are refracted towards the earth. This can be done accurately by calculating $\frac{dk}{dh}$, which is found to be equal to $\frac{dk'}{dh}$ $(=31\cdot4\times10^{-4})$, for $h=$ 0·825 km.

The calculation can be repeated for other initial temperatures; *e.g.*, if we start from $T=296°$, we get the values of Table IV. The thickness of the layer in which the refraction is such as to give waves directed towards the surface of the earth is only about 350 metres under these conditions. Thus we rapidly reach a temperature for which only the air layer directly above the water refracts the whole amount of the electromagnetic

Table IV.

h	$T°k$	P	e	Specific moisture.	d moist air.	A water vapour.	k moist air.	k'
0	296·0	760·0	16·68	13·79	0·001183	16·18	1·001325	[1·001325]
0·5	295·0	717·6	13·76	12·05	0·0011122	16·27	1·001166	1·001167
1·0	294·0	677·3	11·09	10·29	0·001063	16·36	1·001017	1·001010
2·0	290·5	602·7	7·19	7·50	0·000956	16·67	1·000791	1·000675

waves towards the earth. Assuming that the results calculated would exactly correspond to the reality, we could state that in the tropical and subtropical regions, where the temperature of the atmosphere close to the water remains above 292°-293°, the conditions of the lower layers are such that the totality of the waves are refracted downwards. Thus we come to the conclusion that the electric "space" waves, instead of being lost for the major part before they reach a distant receiving antenna—as it was thought by Kiebitz—can, under favourable conditions, be totally refracted downwards; and under less favourable temperature conditions the greater part of them still reaches the receiver.

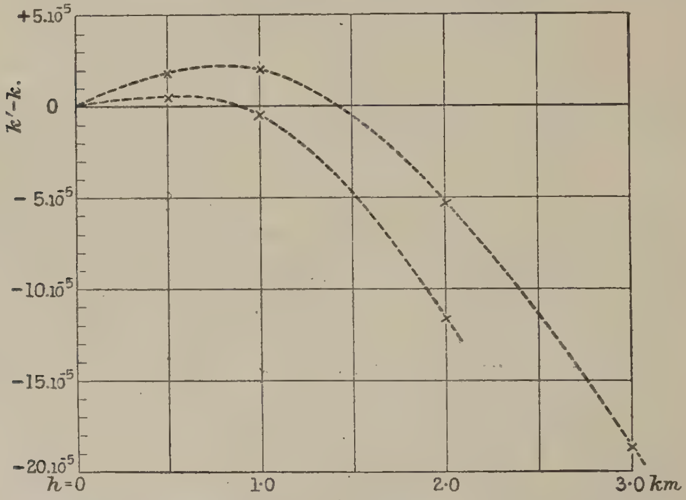

Diagram III.—Difference $K'-K$ at Different Heights (h in Kilometre) and for Different Temperature Conditions.

But the actual figures arrived at can only be approximate, for (*a*) they depend upon the rather uncertain value of k for water-vapour; (*b*) they refer to ideal conditions, without regard to local changes (temperature, wind, clouds, rain, &c.). With regard to (*a*) it may again be emphasised that by employing a larger k-value for water-vapour—which would very likely better correspond to reality—the area above the oceans in which electric waves are totally refracted would extend to the temperate, and possibly even to arctic regions. With regard to (*b*) the discussion of local conditions would lead us to rather vague results as long as the point (*a*) is not settled; at any rate, it would only be worth while discussing the influence of local factors in connection with actual experimental data, which are lacking at present.*

Yet certain general conclusions can already be drawn; *e.g.*, since the average temperature above the oceans varies by less than 1° within 24 hours, the changes of intensity observed at the receiver, when comparing day with night, can hardly be the result of changes of the temperature and corresponding variations of the hygrometric state of the atmosphere.

Similar calculations could be made for the propagation of electric waves above the land. In this case the prismatic effect of the atmosphere is less important, since the relative amount of "space" waves is smaller (*see* Sommerfeld, *loc. cit.*). We shall not attempt any general calculation, because the temperature and other physical conditions are far less constant than above the oceans.

In conclusion, I think that, next to the ionisation of air, the variation of its electric refractive index—due principally to its hygrometric state—constitutes an important factor in the propagation of electromagnetic "space" waves and the intensity at a receiving wireless station.

ABSTRACT.

The author discusses the probable influence of moisture in the atmosphere on the refraction of electromagnetic waves round the earth's surface. The conclusion of Kiebitz that the presence of moisture does not affect the dielectric constant by more than 10 per cent. is shown to be erroneous, being based on the assumption that the Clausius-Mossoti formula, $\frac{k-1}{k+2} \cdot \frac{1}{d} = \text{constant}$, holds when passing from the liquid to the gaseous state. Examples are quoted to show

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* A serious effort to set up regular and accurate measurements in connection with wireless telegraphy has been started recently by an international committee, known under the name "Télégraphie sans fil scientifique" (T.S.F.S.), which has its central seat in Brussels. It had hardly started its activity when the work was interrupted by the war. This Paper is part of the work I have done in that laboratory, with a grant from the "Institut International de Physique Solvay," which I gladly acknowledge.

that this law fails in many cases, especially where the dielectric constant is high in the liquid state.

In the absence of more accurate data for ordinary temperatures, the author prefers to assume a value for the dielectric constant of water vapour obtained by extrapolating the results obtained by Baedeker for higher temperatures. The extrapolated value is almost certainly too low.

From this result, and the average conditions of the atmosphere over the ocean with regard to temperature gradient, &c., deduced from meteorological data, it is shown that the lowest layers of the atmosphere (1,000 to 1,500 metres approximately in depth) refract electromagnetic waves towards the earth, so that the greater part of the space waves will reach the receiver, contrary to the conclusion of Kiebitz.

DISCUSSION.

Dr. C. CHREE (communicated remarks): The author, presumably unaware of their existence, does not refer to the somewhat numerous upper air data which have been published in this country by the Meteorological Office. A study of these data as to the temperature gradients would I think, have proved useful. Inversions are not confined to the lowest layers, but in these layers they are exceptional, and do not suffice to reduce the average temperature gradient to such low values as the author has taken for the first and second kilometres. For these, 5 deg. or 6 deg. per kilometre would not have been too high, especially for tropical regions. If the empirical exponential formula for vapour pressure be assumed, the pressure at any given height varies directly with that at ground level, and so in temperate latitudes is much lower in winter than in summer. It thus seems rather a fundamental point whether wireless phenomena in temperate latitudes show a marked annual variation corresponding with that of vapour pressure at ground level.

The AUTHOR, replying to Dr. Chree, said: A temperature decrease of 5 deg. to 6 deg. in the first kilometre may be or may not be more accurate than the decrease of 2 deg. which I have proposed; the scanty available observations above the oceans only allow us to state that both figures are probable values for the yearly average of the temperature gradient. If there were serious reasons to prefer the figures proposed by Dr. C. Chree, it would simply mean that the height of the air layer within which total refraction occurs is smaller than it appears from any calculations; but the order of magnitude of the phenomenon remains the same. The question of an inversion of the temperature gradient in upper atmospheric layers has not been considered in this connection. For large h's the waves are bent apart from the surface of the earth (see my calculations), and even a strong negative gradient is not likely to determine the bend in the other direction. It would be very interesting to verify how far a relation between vapour pressure and annual variation at a wireless receiver—as well as many other relations—is verified by actual observations. But, as far as the wireless phenomena are concerned, there are practically no data available at present. At the eve of starting such measurements it was precisely interesting to try to find out which could possibly be the meteorological factors most likely to affect the propagation of electromagnetic waves; and whatever may be disputed now or later with regard to the actual numbers given in my Paper, I think it will remain true that the amount of water vapour present in the atmosphere is one of the important factors.

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